Set-membership identifiability and application to fault detection and diagnosis

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Definitions and methods for checking identifiability of linear and nonlinear classical systems are now well established.
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But, what if the system is not identifiable?
Example: \[
\begin{aligned}
\dot{x}_1 &= x_1 + t \cos(\varpi), \\
{x}_1(0) &= \varepsilon.
\end{aligned}
\]
Solution: \[
x_1(t) = (-1 - t + e^t) \cos(\varpi) + \varepsilon e^t.
\]
Hypothesis: \(\mathcal{U}_p = [0, 2\pi], \ \varpi \in P^* = \left[\frac{\pi}{2}, \frac{3\pi}{2}\right].\)
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Motivation

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Motivation

- It may be the case that there exists a partition of the parameter space into connected subsets so that every subset can be associated with a distinguishable output behavior.
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→ set-membership identifiability (SM-identifiability)
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So, why not use set-membership methods? → set-membership identifiability (SM-identifiability)

Two definitions of global SM-identifiability that we have proposed (Jauberthie et al., IFAC WC 2011) are recalled:

- a conceptual definition,
- a definition relying on a measure $\mu$ (can be put in correspondence with operational set-membership estimation methods).

An FDI method relying on SM-identifiability is presented.
We consider the uncertain system:

\[
\Gamma_1^P = \begin{cases} 
\dot{x}(t, p) = f(x(t, p), u(t), p), \\
y(t, p) = h(x(t, p), p), \\
x(t_0, p) = x_0 \in X_0, \\
p \in P \subset \mathcal{U}_P, \\
t_0 \leq t \leq T,
\end{cases}
\]

(1)

where:

- \(x(t, p) \in \mathbb{R}^n\): state variables at time \(t\),
- \(y(t, p) \in \mathbb{R}^m\): output vector at time \(t\),
- \(u(t) \in \mathbb{R}^r\): input vector at time \(t\),
- \(x_0 \in X_0, X_0\): a bounded set,
- \(f, h\): real functions, analytic on \(M\) (an open set of \(\mathbb{R}^n\)),
- \(p \in P \subset \mathcal{U}_P\): vector of parameters, \(\mathcal{U}_P \subset \mathbb{R}^p\): an a priori known set of admissible parameters.
**Conceptual definitions**

**Notation:** $Y(P, u)$ (respectively $Y(P)$ when $u = 0$): the set of outputs, solution of $\Gamma_P^1$

- Global SM-identifiability

**Definition: Case of controlled systems**

The model $\Gamma_P^1$ given by (1) is **globally SM-identifiable** for $P^* \neq \emptyset$, $P^* \subset \mathcal{U}_P$, if there exists an input $u$ such that $Y(P^*, u) \neq \emptyset$ and $Y(P^*, u) \cap Y(\bar{P}, u) \neq \emptyset$, $\bar{P} \subset \mathcal{U}_P \implies P^* \cap \bar{P} \neq \emptyset$. 
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- Extension to local SM-identifiability for \( P^* \).
Example: \( \dot{x}_1 = x_1 + t \cos(\varpi) \), \( x_1(0) = \varepsilon \).

Globally SM-identifiable:

\[
\mathcal{U}_P = [0, 2\pi], \quad P^* = \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] = P^*_1
\]
Example: \( \dot{x}_1 = x_1 + t \cos(\omega) \), \( x_1(0) = \varepsilon \).

Globally SM-identifiable:

\( \mathcal{U}_P = [0, 2\pi], \ P^* = \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] = P_1^* \)
**Example:** \( \dot{x}_1 = x_1 + t \cos(\varpi), \ x_1(0) = \varepsilon. \)

**Not globally SM-identifiable:**

\[ \mathcal{U}_P = [0, 2\pi], \ P^* = \left[ \pi, \frac{3\pi}{2} \right] = P_2^* \subset P_1^* \]
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Not globally SM-identifiable:
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\mathcal{U}_P = [0, 2\pi], \quad P^* = \left[ \pi, \frac{3\pi}{2} \right] = P_2^*
\]
A model can be SM-identifiable for $P_1^*$ and not SM-identifiable for $P_2^* \subset P_1^*$.

Important to develop a definition of identifiability in which $P^*$ can be taken as small as desired.

$\rightarrow \mu$-SM-identifiability.
**μ-SM-identifiability**

Let us now consider a bounded set $\Pi$ of $\mathbb{R}^p$.

$\mu(\Pi) = \text{diameter}$ of $\Pi$ is the least upper bound of 

$\{d(\pi_1, \pi_2), \pi_1, \pi_2 \in \Pi\}$, with $d$ a classical metric on $\mathbb{R}^p$.

If $\Pi$ is not bounded, $\mu(\Pi) = +\infty$.

$P^*$ is supposed to be bounded.
Set-membership identifiability

Motivation

\[ \mu \text{-SM-identifiability} \]

Let us now consider a bounded set \( \Pi \) of \( \mathbb{R}^p \).

\[ \mu(\Pi) = \text{diameter of } \Pi \text{ as the least upper bound of } \{d(\pi_1, \pi_2), \pi_1, \pi_2 \in \Pi\}, \text{ with } d \text{ a classical metric on } \mathbb{R}^p. \]

If \( \Pi \) is not bounded, \( \mu(\Pi) = +\infty \).

\( P^* \) is supposed to be bounded.

**Definition**

The model \( \Gamma^P_1 \) given by (1) is **globally \( \mu \)-SM-identifiable for \( P^* \neq \emptyset \)**, \( \mu(P^*) \) as small as desired, if there exists an input \( u \) such that

- \( Y(P^*, u) \neq \emptyset \) and
- \( Y(P^*, u) \cap Y(\bar{P}, u) \neq \emptyset \), \( \bar{P} \subset \mathcal{U}_P \implies P^* \cap \bar{P} \neq \emptyset \).

If \( \mu(P^*) \geq \varepsilon \), then we refer to **\( \varepsilon \)-SM-identifiability**.

\( \Rightarrow \) **Practical importance of \( \varepsilon \)-SM-identifiability** because \( \varepsilon \) defines the granularity at which identifiability is considered.
Method: based on differential algebra (Kolchin and al., 1973)

- elimination order \( \{p\} < \{y, u\} < \{x\} \)
  \((\Rightarrow \text{eliminate unobservable state variables}),\)
- Rosenfeld-Groebner algorithm = elimination algorithm,
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\[ \Rightarrow \text{relations between inputs, outputs and parameters:} \]

\[ w_i(y, u, p) = m_0(y, u) + \sum_{k=1}^{n_i} \gamma_k^i(p)m_k(y, u), \ i = 1, \ldots, m \]

\[ \rightarrow (\gamma_k^i)_{1 \leq k \leq l} \text{ are rational in } p, \gamma_u^i \neq \gamma_v^i \ (u \neq v), \]

\[ \rightarrow (m_k)_{1 \leq k \leq l} \text{ are differential polynomials with respect to } y \text{ and } u \text{ and } m_0 \neq 0. \]
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Size of the system : \( m = \text{number of observations.} \)
Injectivity of a function (Lagrange and al., 2007)

Consider a function \( f : \mathcal{A} \rightarrow \mathcal{B} \) and any set \( \mathcal{A}_1 \subseteq \mathcal{A} \). The function \( f \) is said to be a partial injection of \( \mathcal{A}_1 \) over \( \mathcal{A} \) (resp. restricted-partial injection), noted \( (\mathcal{A}_1, \mathcal{A}) \)-injective (resp. \( (\mathcal{A}_1, \mathcal{A}) \)-restricted-injective), if,

\[
\forall a_1 \in \mathcal{A}_1, \forall a \in \mathcal{A}, a_1 \neq a \Rightarrow f(a_1) \neq f(a)
\]

(resp. \( \forall a_1 \in \mathcal{A}_1, \forall a \in \mathcal{A}_1^c, f(a_1) \neq f(a) \)).

\( f_1 \) and \( f_2 \) are \( ([x_1] ; [x]) \)-restricted-injective and \( ([x_1] ; [x]) \)-injective. 
\( f_3 \) is \( ([x_1] ; [x]) \)-restricted-injective but not \( ([x_1] ; [x]) \)-injective. 
\( f_4 \) is not \( ([x_1] ; [x]) \)-injective.
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(resp. $\forall a_1 \in \mathcal{A}_1, \forall a \in \mathcal{A}_1^c, f(a_1) \neq f(a)$).

→ S. Lagrange has developed an algorithm based on interval analysis for testing the injectivity of a given differentiable function (definition in red colour) (solver ITVIA: Injectivity Test Via Interval Analysis).

→ We propose an algorithm based on ITVIA to test the definition of restricted-partial injection and obtain the set-membership sets.
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Remarks: For simplify: $m = 1$ and $w_1 = w_0$,

$$w_0(y, u, p) = m_0(y, u) + \sum_{k=1}^{n} \gamma_k(p)m_k(y, u).$$

$l = the$ higher order derivative of $y$ in $w_0$
Proposition

Suppose that \( \triangle w_0(y) = \det(m_k(y, u), k = 1, \ldots, n) \neq 0. \)
Let \( P^* \) a subset of \( \mathcal{U}_P \) and the function \( \phi : p = (p_1, \ldots, p_p) \mapsto (\gamma_1(p), \ldots, \gamma_n(p), y(t_0^+, p), \ldots, y^{(l-1)}(t_0^+, p)). \)

If \( \phi \) is \((P^*, \mathcal{U}_P)-restricted-injective\), then the model \( \Gamma_1^P \) is globally SM identifiable for \( P^* \).

If \( \phi \) is \((P^*, \mathcal{U}_P)-injective\) then the model is \( \mu \)-SM identifiable for \( P^* \).

In the two cases, if the coefficient of \( y^{(l)} \) in \( w_0 \) is not equal to 0 at \( t_0 \), then the reciprocal is valid.
\[
\begin{aligned}
\dot{x}_1 &= p_1 x_1^2 + \sin(p_2) x_1 x_2, \quad x_1(0) = 1 \\
\dot{x}_2 &= p_3 x_1^2 + x_1 x_2, \quad x_2(0) = b \\
y &= x_1. 
\end{aligned}
\]

(\(p_1, p_2, p_3\)) \in \mathcal{U}_P = \mathbb{R} \times [0, 2\pi] \times \mathbb{R}^+ : \text{unknown parameters.}

Let \(p_4 = \sin(p_2)\).
\[ \begin{cases} \dot{x}_1 = p_1 x_1^2 + \sin(p_2) x_1 x_2, & x_1(0) = 1 \\ \dot{x}_2 = p_3 x_2^2 + x_1 x_2, & x_2(0) = b \\ y = x_1. \end{cases} \] (2)

\((p_1, p_2, p_3) \in \mathcal{U}_P = \mathbb{R} \times [0, 2\pi] \times \mathbb{R}^+ : \text{unknown parameters.}\)

Let \(p_4 = \sin(p_2).\)

The Rosenfeld-Groebner algorithm in Maple gives 3 cases:

- an impossible case: \(y = 0\) because \(y(0) = 1,\)
- an particular case \(p_4 = 0\) (hence \(p_2 = 0\) or \(\pi\))
- a polynomial:
  \[ w_0(y) = \dot{y}^2 - y\ddot{y} + \dot{y}y^2 + p_1(\dot{y}y^2 - y^4) + p_4p_3y^4. \]
  \[ \triangle w_0(y) = 2y^5\dot{y}^2 - y^6\ddot{y} \not\equiv 0. \]

Let \(\phi : (p_1, p_2, p_3) \rightarrow (p_1, \sin(p_2)p_3, p_1 + \sin(p_2)b).\)
\[
\begin{aligned}
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Let \(p_4 = \sin(p_2)\).

\(\phi : (p_1, p_2, p_3) \rightarrow (p_1, \sin(p_2) p_3, p_1 + \sin(p_2) b)\): 
\(\rightarrow\) is \((P^*, \mathcal{U}_P)\)-restricted-injective:

\(\forall p^*_2 \in ]0, \pi[, \forall \bar{p}_2 \in ]\pi, 2\pi[, \sin(p^*_2) > 0 \text{ and } \sin(\bar{p}_2) < 0,\)
\(\rightarrow\) is not \((P^*, \mathcal{U}_P)\)-injective for \(P^* = \mathbb{R} \times ]0, \pi[ \times \mathbb{R}^+ :\)

the function \(\sin\) is not injective on \(]0, \pi[.\)

**Conclusion:**
The model is globally SM-identifiable for \(P^* = \mathbb{R} \times ]0, \pi[ \times \mathbb{R}^+.\) and it is not \(\mu\)-SM-identifiable for \(P^* = \mathbb{R} \times ]0, \pi[ \times \mathbb{R}^+.\)
FDI method illustrated on an example based on SM-identifiability

\[
\begin{aligned}
\dot{x}_1 &= \alpha_1 (x_2 - x_1) - \frac{V_m x_1}{1 + x_1}, \\
\dot{x}_2 &= \alpha_2 (x_1 - x_2), \\
x_1(0) &\in [0.62, 0.63], \ x_2(0) = 0, \\
y &= x_1.
\end{aligned}
\]

This model represents the capacity of the macrophage mannose receptor to endocytose soluble macromolecule,

- \(x_1\) (resp. \(x_2\)) is the enzyme concentration outside (resp. inside) the macrophage,

\(p = (\alpha_1, V_m, \alpha_2) \in [0, +\infty] \times [0, +\infty] \times [0, +\infty]: \text{the unknown parameters vector.}\)
Checking SM-identifiability:
The package diffalg of Maple gives the following output polynomial:
\[ w_0(y) = \ddot{y}(1 + y)^2 + \gamma_1 \dot{y}(1 + y)^2 + \gamma_2 y(1 + y) + \gamma_3 \dot{y}, \]
where \( \gamma = \{\alpha_1 + \alpha_2, \alpha_2 V_m, V_m\} \).

→ First hypothesis:
Maple \( \Rightarrow \Delta w_0(y) \neq 0. \)

→ Second hypothesis:
For all \( P^* \subset [0, +\infty] \times [0, +\infty] \times [0, +\infty], \)
\( \phi : (\alpha_1, \alpha_2, V_m) \rightarrow (\alpha_1 + \alpha_2, \alpha_2 V_m, V_m) \) is \((P^*, \mathbb{R}^3)\)-injective.

The system is \( \mu \)-SM-identifiable for all \( P^* \subset [0, +\infty] \times [0, +\infty] \times [0, +\infty]. \)

→ This guarantees that the solution set for \( \gamma \) reduces to one connected set.
Simulations:

\[
\begin{cases}
\dot{x}_1 = \alpha_1 (x_2 - x_1) - \frac{V_m x_1}{1 + x_1}, \\
\dot{x}_2 = \alpha_2 (x_1 - x_2), \\
x_1(0) \in [0.62, 0.63], \ x_2(0) = 0,
\end{cases}
\]

\( y = x_1, \) \hspace{1cm} (4)

→ Simulation in Matlab with normal values: \( \alpha_1 = 0.011, \)
\( \alpha_2 = 0.02 \) and \( V_m = 0.1, \)

→ \( y(t) = \bar{y}(t) + \eta(t), \) \( \bar{y} \) is the exact output corresponding to the exact value of parameters, \( \eta(t) \) = truncated gaussian noise such that \( \eta(t) \in [-0.001, 0.001]. \)

→ Observations done at discrete times \((t_j)_{j=1,...,N}\) on the interval \([0, 117]\) with a sampling equal to \( \frac{1}{2}. \)
Parameter estimation and FDI:

\( w_0(y) = \ddot{y}(1 + y)^2 + \gamma_1 \dot{y}(1 + y)^2 + \gamma_2 y(1 + y) + \gamma_3 \dot{y}, \)

with \( \gamma_1 = \alpha_1 + \alpha_2, \gamma_2 = \alpha_2 V_m \) and \( \gamma_3 = V_m. \)

\( \rightarrow \) \( y_p(t_j) \) (resp. \( y_{pp}(t_j) \)) the estimate of \( \dot{y}(t_j) \) (resp. \( \ddot{y}(t_j) \)) obtained by a finite differences method extended to interval analysis,

\( \rightarrow \) Problem : Find \( \gamma \) such that \( 0 \in w_0(y) \)

\( \rightarrow \) System which has to be solved: \([A][\gamma] = [b]\)

where \([A]_j = ([y_p(t_j)(1 + y(t_j))^2], [y(t_j)(1 + y(t_j))], [y_p(t_j)])\)

and \([b]_j = [-y_{pp}(t_j)(1 + y(t_j))^2],\)

\( \rightarrow \) Resolution done with SIVIA.
Case of nominal behaviour
Initial intervals: $[\gamma_1] = [0, 0.04]$, $[\gamma_2] = [0, 0.003]$, $[\gamma_3] = [0, 0.2]$,
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1st case:

→ Bisection precision $\varepsilon = 0.001$,

→ We obtain in 14.18 seconds:
  $[\alpha_1] = [0, 0.0401]$, $[\alpha_2] = [0, 0.0437]$ and $[V_m] = [0.06875, 0.13203]$.

2nd case:

→ Bisection precision $\varepsilon = 0.0001$,

→ We obtain in 177.55 seconds:
  $[\alpha_1] = [0, 0.0329]$, $[\alpha_2] = [0.0071, 0.0317]$ and $[V_m] = [0.094824, 0.10527]$.

All these intervals contain the normal values $\alpha_1 = 0.011$, $\alpha_2 = 0.02$ and $V_m = 0.1$. 
Case of a fault on $\alpha_2$

We suppose $\alpha_2 = 2$ (instead of 0.02).

Initial intervals: $[\gamma_1] = [0, 3]$, $[\gamma_2] = [0, 1]$, $[\gamma_3] = [0, 0.2]$,

$\rightarrow$ Bisection precision $\varepsilon = 0.001$,

$\rightarrow$ We obtain in 25.15 minutes:

$[\alpha_1] = [0.0000, 0.5050]$, $[\alpha_2] = [1.1200, 10.4435]$ and $[V_m] = [0.0242, 0.1790]$.

The real faulty value of $\alpha_2$ is contained in the estimated interval for $\alpha_2$.

The fault is detected and localised.
Case of a fault on $\alpha_2$

We suppose $\alpha_2 = 1.5$ at $t = 17s$.

Initial intervals: $[\gamma_1] = [0, 0.04]$, $[\gamma_2] = [0, 0.003]$, $[\gamma_3] = [0, 0.2]$,

$\rightarrow$ Bisection precision $\varepsilon = 0.05$,

$\rightarrow$ The fault is detected 2.59s after its occurrence.

Initial intervals: $[\gamma_1] = [0, 2]$, $[\gamma_2] = [0, 0.18]$, $[\gamma_3] = [0, 0.114]$, We obtain in 23s:

$[\alpha_1] = [0.0000, 0.9039]$, $[\alpha_2] = [0.8771, 2.4794]$ and $[V_m] = [0.0726, 0.1400]$.

- The real faulty value of $\alpha_2$ is contained in the estimated interval for $\alpha_2$.
- The fault is localised.
Case of a fault on $V_m$

We suppose $V_m = 0.15$ at $t = 22s$.

Initial intervals: $\gamma_1 = [0, 0.04]$, $\gamma_2 = [0, 0.003]$, $\gamma_3 = [0, 0.2]$, 

$\rightarrow$ Bisection precision $\varepsilon = 0.05$,

$\rightarrow$ A fault is detected 1.01s after its occurence.

Initial intervals: $\gamma_1 = [0, 2]$, $\gamma_2 = [0, 0.18]$, $\gamma_3 = [0, 0.114]$, We obtain in 23s:

$[\alpha_1] = [0.0000, 0.0399]$, $[\alpha_2] = [0.000, 0.0473]$ and $[V_m] = [0.1271, 0.1985]$.

The real faulty value of $V_m$ is contained in the estimated interval for $V_m$.

The fault is localised.
Conclusion:

- Definitions of SM-identifiability / \( \mu \)-SM-identifiability
  - provide a way to study identifiability for uncertain bounded-error systems,
  - provide the guaranty that two situations corresponding to different uncertain parametrized setting are distinguishable.
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- Method to analyze ($\mu$-)SM-identifiability,

- FDI method relying on a parameter estimation scheme built on the analysis of identifiability
  → guarantees that the solution set reduces to one connected set, avoiding this way the pessimism of SM methods.

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