Mixed Integer Nonlinear Programming for Aircraft Conflict Avoidance

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Outline

1. Introduction

2. MINLP for aircraft conflict avoidance
   - Heading angle change model
   - Numerical results
   - Maximizing the number of solved conflicts using velocity change
   - Algorithm for aircraft conflict avoidance
   - Numerical results

3. Conclusions and Perspectives
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3 Conclusions and Perspectives
Air Traffic Management

- Air traffic management aims at ensuring smooth running of the transportation system under safety while keeping flights on schedule.

- Growth of Air Traffic
  - Traffic volumes have experienced continuous and sustained growth.
  - Number of 2012 flights: 31,177,541.
  - Air traffic is expected to increase by a factor of two or three in the coming two decades.

- Promote automation
  - SESAR (En-Route Air Traffic Soft Management Ultimate System) → European project
  - NextGen (Next Generation Air Transportation System) → US project
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Safety issue
Aircraft conflict

- During their flights, aircraft have to maintain safety.
  - Aircraft have to maintain a minimum separation between each other during the entire flight.
- Any violation of this minimum separation is defined as a conflict.

- Minimum required separation
  - Horizontal: 5NM (1NM (nautical mile) = 1, 852m)
  - Vertical: 1, 000ft (1ft (feet) = 0.3048m)
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  ⇒ Aircraft have to maintain a minimum separation between each other during the entire flight.
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Minimum required separation

- Horizontal: 5NM (1NM = 1,852m)
- Vertical: 1,000ft (1ft = 0.3048m)

Aircraft separation maneuvers

- Heading angle change (HAC)
- Altitude change (AC)
- Velocity change (VC)
State of the Art

- Over the past 15 years, many mathematical models for aircraft conflict avoidance have been proposed.
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Optimization problem

**Objective:** Minimize path deviations from the original path

**Variables:** Angle deviations

**Constraints:** Minimum separation condition for each pair of aircraft
Bounds on heading angle changes
Modeling separation constraints (1/3)

- **A**: a set of aircraft sharing the same airspace.
- Each aircraft \( i \in A \) is initially identified by the triplet \((x_i^0, y_i^0, \Phi_i)\) giving the abscissa, ordinate and direction of motion in \([-\pi, \pi]\).

**Figure**: Before deconfliction
Modeling separation constraints (1/3)

- $A$: a set of aircraft sharing the same airspace.
- Each aircraft $i \in A$ is initially identified by the triplet $(x^0_i, y^0_i, \Phi_i)$ giving the abscissa, ordinate and direction of motion in $]-\pi, \pi[$.

![Diagram with aircraft positions and directions](image)

**Figure**: After deconfliction
Modeling separation constraints (2/3)

- The abscissa and ordinate of an aircraft \(i\) at a time \(t\) are given by
  \[
  x_i(t) = x_i^0 + \cos(\Phi_i + \theta_i)v_it \\
  y_i(t) = y_i^0 + \sin(\Phi_i + \theta_i)v_it.
  \]

- The necessary minimum separation distance condition between two aircraft \(i\) and \(j\) at the same flight level can be expressed as follows:
  \[
  (x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2 \geq d^2, \quad \forall \ t \geq 0.
  \]

- By substituting (1) and (2) into (3), we get
  \[
  t^2\|V_{ij}^r\|^2 + 2t \ X_{ij}^{0r} \cdot V_{ij}^r + \|X_{ij}^{0r}\|^2 - d^2 \geq 0, \quad \forall \ t \geq 0.
  \]

\[V_{ij}^r := \begin{pmatrix}
  \cos(\Phi_i + \theta_i)v_i - \cos(\Phi_j + \theta_j)v_j \\
  \sin(\Phi_i + \theta_i)v_i - \sin(\Phi_j + \theta_j)v_j
\end{pmatrix}\] is the relative speed

\[X_{ij}^{0r} := \begin{pmatrix}
  x_i^0 - x_j^0 \\
  y_i^0 - y_j^0
\end{pmatrix}\] is the relative initial distance
The abscissa and ordinate of an aircraft $i$ at a time $t$ are given by

$$
\begin{align*}
    x_i(t) &= x_i^0 + \cos(\Phi_i + \theta_i)v_i t \\
    y_i(t) &= y_i^0 + \sin(\Phi_i + \theta_i)v_i t.
\end{align*}
$$

The necessary minimum separation distance condition between two aircraft $i$ and $j$ at the same flight level can be expressed as follows:

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$V_{ij}^r := \left( \begin{array}{c}
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→ Use the technique proposed by Cafieri and Durand – JOGO, 2014.
Modeling separation constraints (3/3)

- Separation condition

\[ t^2 \| V_{ij}^r \|^2 + 2t X_{ij}^{0r} \cdot V_{ij}^r + \| X_{ij}^{0r} \|^2 - d^2 \geq 0, \quad \forall \ t \geq 0. \]  

\[ \implies \] a quadratic equation in one unknown \( t \)

- By differentiating with respect to \( t \), the minimum is reached for

\[ t_{ij}^m = - \frac{X_{ij}^{0r} \cdot V_{ij}^r}{\| V_{ij}^r \|^2}. \]

- By substituting this into (4), we obtain the following separation condition

\[ \| V_{ij}^r \|^2 (\| X_{ij}^{0r} \|^2 - d^2) - (X_{ij}^{0r} \cdot V_{ij}^r)^2 \geq 0. \]

\[ \implies \] no longer depends on \( t \)

- We introduce an auxiliary binary variable \( y_{ij} \) for each pair of aircraft \( i \) and \( j \) such that

\[ y_{ij} = \begin{cases} 
1 & \text{if } t_{ij}^m \geq 0, \\
0 & \text{otherwise}. 
\end{cases} \]

- This yields the following separation condition

\[ y_{ij} \left( \| V_{ij}^r \|^2 (\| X_{ij}^{0r} \|^2 - d^2) - (X_{ij}^{0r} \cdot V_{ij}^r)^2 \right) \geq 0. \]
MINLP formulation: HAC (1/2)

Variables:

- \( \theta_i, \ (\geq \theta_{\text{min}}, \leq \theta_{\text{max}}), \ \forall i \in A \) angle variation of aircraft \( i \) (continuous)
- \( V_{ij}^2, \ \forall (i, j) \in A, i < j \) square of the relative velocity of \( i \) and \( j \) (continuous)
- \( p_{ij}, \ \forall (i, j) \in A, i < j \) inner product \( X_{ij}^0 \cdot V_{ij}^r \) (continuous)
- \( t_{ij}^m, \ \forall (i, j) \in A, i < j \) minimum time for separation (continuous)
- \( y_{ij}, \ \forall (i, j) \in A, i < j, \) used to check if \( t_{ij}^m \geq 0 \) (binary)

Objective:

\[
\min \sum_{i \in A} \theta_i^2
\]

minimizing angle deviations
Constraints:

- definition of $V_{ij}^{2r}$ (trigonometric functions)
  \[
  V_{ij}^{2r} = (v_i \cos(\phi_i + \theta_i) - v_j \cos(\phi_j + \theta_j))^2 
  + (v_i \sin(\phi_i + \theta_i) - v_j \sin(\phi_j + \theta_j))^2 
  \forall i, j \in A, i < j
  \]

- inner product in the separation condition (trigonometric functions)
  \[
  p_{ij} = (x_i^0 - x_j^0) (v_i \cos(\phi_i + \theta_i) - v_j \cos(\phi_j + \theta_j)) 
  + (y_i^0 - y_j^0) (v_i \sin(\phi_i + \theta_i) - v_j \sin(\phi_j + \theta_j)) 
  \forall i, j \in A, i < j
  \]

- definition of $t_{ij}^m$ (bilinear, trigonometric functions)
  \[
  t_{ij}^m V_{ij}^{2r} + p_{ij} = 0 
  \forall (i, j) \in A, i < j
  \]

- check sign of $t_{ij}^m$ (bilinear with binary var.)
  \[
  t_{ij}^m (2y_{ij} - 1) \geq 0 
  \forall (i, j) \in A, i < j
  \]

- separation (quadratic, product with binary var.)
  \[
  y_{ij} \left( (\|X_{ij}^0\|^2 V_{ij}^{2r}) - (p_{ij})^2 - ((d)^2 V_{ij}^{2r}) \right) \geq 0 
  \forall (i, j) \in A, i < j
  \]
Heading angle changes model

HAC model

\[
\begin{align*}
\text{min} & \quad \sum_{i \in A} \theta_i^2 \\
\text{s.t.} & \quad \theta_{\text{min}} \leq \theta_i \leq \theta_{\text{max}} & \forall i \in A \\
& \quad t_{ij}^m V_{ij}^{2r} + p_{ij} = 0 & \forall i, j \in A, i < j \\
& \quad t_{ij}^m (2y_{ij} - 1) \geq 0 & \forall i, j \in A, i < j \\
& \quad y_{ij} \left( V_{ij}^{2r} \left( \|X_{ij}^0\|_2^2 - d^2 \right) - p_{ij}^2 \right) \geq 0 & \forall i, j \in A, i < j \\
& \quad y_{ij} \in \{0, 1\} & \forall i, j \in A, i < j \\
\end{align*}
\]

\[\Rightarrow\] MINLP model
Returning to the initial trajectory

- The idea proposed by Alonso-Ayuso, Escudero and Martín-Campo, 2014 can be easily applied in our context.
- Solve a quadratic programming (QP) model for each pair of aircraft \((i, j)\).
- Knowing \(\theta_i^*\) for aircraft \(i\), the new coordinates of aircraft \(i \in A\) are given by
  \[
  x_i(t) = x_i^0 + \cos(\Phi_i + \theta_i^*)v_i t \quad \text{and} \quad y_i(t) = y_i^0 + \sin(\Phi_i + \theta_i^*)v_i t.
  \]
- The quadratic model to be solved for each pair \((i, j)\) is
  \[
  \min_{t_{ij}} \left\| (x_i(t_{ij}) - x_j(t_{ij})) \right\|^2.
  \]
  (6)
  
  \(\rightarrow\) Let \(t_{ij}^*\) the optimal solution.
  
  \(\rightarrow\) Compute \(T_i^* := \max_{j \neq i, j \in A} t_{ij}^*\).
  
  \(\rightarrow\) This time corresponds to the point \((x_i(T_i^*), y_i(T_i^*))\).
Returning to the initial trajectory

\[(x_i(T_i^*), y_i(T_i^*))\]

Figure: Forcing aircraft to return to their initial trajectories
Testing environment

- Both models are formulated using AMPL.
- 20 randomized circle problems.

**HAC model**

Global solution

*spatial Branch-and-Bound*

**COUENNE 0.4** (Belotti et al., 2008)

**QP model**

Interior point algorithm

**IPOPT 3.11** (Wächter and Biegler, 2006)

Test problems

- $n \in \{2, \ldots, 5\}$ aircraft
- $n(n-1)/2$ conflicts
- $d = 5$ NM, $v = 400$ NM/h
Numerical results: HAC

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<th>$n$</th>
<th>$n_c$</th>
<th>$n_{rc}$</th>
<th>time (s)</th>
<th>obj</th>
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</table>
Conflict avoidance using velocity change

Velocity change maneuver

- Not sufficiently studied before 2000 (according to Yang-Kuchar’s survey – 2000).
- Extensively studied since 2000.

Subliminal control

- Subliminal control is a novel concept for computer-aided air traffic management. ⇒ introduced in the context of ERASMUS project
- The aim of the automated subliminal control system is to carry out small adjustments in the speeds of aircraft early enough to prevent the perception of a risk of conflict by the air traffic controller.
- These actions have to be small so as to be imperceptible by the air traffic controller. ⇒ [-6%, +3%] of the original velocity
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Maximizing the number of solved conflicts using VC

- **Variables**
  - \( z_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are separated (no conflict)} \forall i, j \in A, i < j \\ 0 & \text{otherwise} \end{cases} \)
  - \( \bar{v}_i, \ (\geq v_{\text{min}}, \leq v_{\text{max}}), \ \forall i \in A \) (continuous).

**Max. VC model**

\[
\begin{align*}
\text{max} \quad & \sum_{(i,j) \in B} z_{ij} \\
\text{s.t.} \quad & v_{\text{min}} \leq \bar{v}_i \leq v_{\text{max}} \quad \forall i \in A \\
& (X_{ij}^0 \cdot \bar{v}_j^r)(2z_{ij} - 1) \geq 0 \quad \forall i, j \in A, i < j \\
\text{OR} \quad & \left( (X_{ij}^0 \cdot \bar{v}_j^r)^2 - ||\bar{v}_j^r||^2 (||X_{ij}^0||^2 - d^2) \right)(2z_{ij} - 1) \leq 0 \\
& z_{ij} \in \{0, 1\} \quad \forall i, j \in A, i < j
\end{align*}
\]

\( \Rightarrow \text{MINLP model} \)
Maximizing the number of solved conflicts using VC

- **Variables**
  - \( z_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are separated (no conflict)} \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \in A, i < j \)
  - \( \bar{v}_i, \ (\geq v_{min}, \leq v_{max}), \quad \forall i \in A \) (continuous).

**Max. VC model**

\[
\text{max} \quad \sum_{(i,j) \in B} z_{ij} \\
\text{s.t.} \quad v_{min} \leq \bar{v}_i \leq v_{max} \quad \forall i \in A \\
(\mathbf{X}_{ij}^{0r} \cdot \bar{v}_i^r)(2z_{ij} - 1) \geq 0 \quad \forall i, j \in A, i < j \\
\text{OR} \quad \left( (\mathbf{X}_{ij}^{r0} \cdot \bar{v}_i^r)^2 - ||\bar{v}_i^r||^2 (||\mathbf{X}_{ij}^{r0}||^2 - d^2) \right)(2z_{ij} - 1) \leq 0 \quad \forall i, j \in A, i < j \\
z_{ij} \in \{0, 1\} \quad \forall i, j \in A, i < j \\
\]

\( \Rightarrow \) MINLP model

- Promising computing time
- Not guaranteed solving all conflicts (head-to-head conflicts, bounds on velocity changes)

Riadh Omheni (ENAC)
MINLP for Aircraft Conflict Avoidance
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Algorithm: Max. VC + HAC

**Algorithm**  Aircraft conflict avoidance by sequentially using VC and HAC

**Require:**  $n$: number of aircraft, $v$: initial velocity, $\phi$: direction of motion

1. Detect all head-to-head conflicts.
2. Solve Max. VC without considering head-to-head conflicts.  \[\Rightarrow\text{a pre-processing step}\]
3. **If** all conflicts are solved **then**
   - Stop.
   **Else**
   - Solve HAC with aircraft having new velocities given by the solution of Max. VC and then go to Step 4.
4. Solve the QP problem.
### Numerical results

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In most cases, pre-processing significantly reduces computing time.
## Numerical results

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In most cases, pre-processing significantly reduces computing time.

Riadh Omheni (ENAC)
MINLP for Aircraft Conflict Avoidance
June, 2015
In most cases, pre-processing significantly reduces computing time.
Performance profile: Total CPU time

- Performance profile of Dolan and Moré.
- Graphical way of comparing both versions of our algorithm.
- 20 randomized circle problems.

Figure: Comparing total CPU time for Algorithm Max. VC + HAC with and without pre-processing on a collection of 20 problems.
Outline

1 Introduction

2 MINLP for aircraft conflict avoidance
   - Heading angle change model
   - Numerical results
   - Maximizing the number of solved conflicts using velocity change
   - Algorithm for aircraft conflict avoidance
   - Numerical results

3 Conclusions and Perspectives
Conclusions and Perspectives

Conclusions

- **New model** for solving aircraft conflicts using HAC maneuver.
- **New algorithm** for aircraft conflict avoidance.
- Benefit of **combining two maneuvers** for aircraft conflict avoidance.
- Very promising numerical results.

Perspectives

- Efficient reformulations of some nonlinearities for the HAC model.
- New heuristics for solving the HAC model.

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⇒
A project founded by French National Agency of Research.
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Thank you for your attention!