Control of Timed Systems

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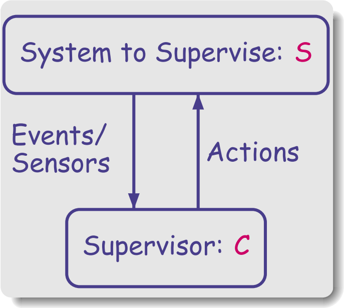
Formalisation des Activités Concurrentes (FAC)
April 3–4, 2008

Toulouse, France

Context: Design of Real-Time Systems



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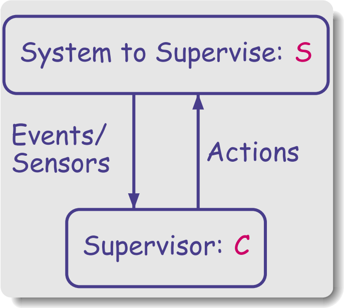
Build Safe Systems



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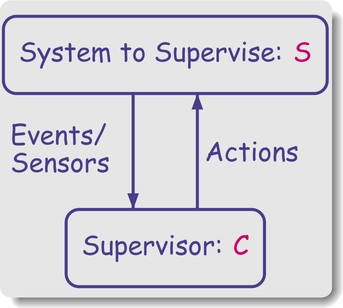
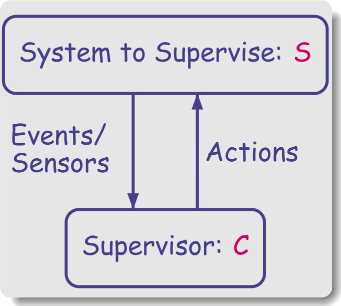


Property φ

Build Safe Systems

Modeling
- Timed Automata
- Time Petri Nets
- Timed Logics



Property φ

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Build Safe Systems

System to Supervise: S

Events/ Sensors

Actions

Supervisor: C

Verification
- Test
- Theorem Proving
- Model-Checking

Property φ

Context: Design of Real-Time Systems

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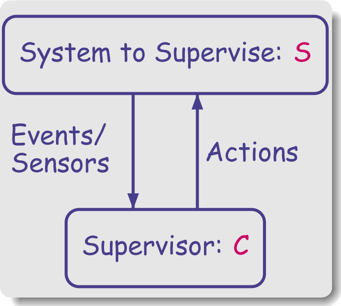
Diagnosis & Control
- Diagnosis
- Control
- Optimal Control



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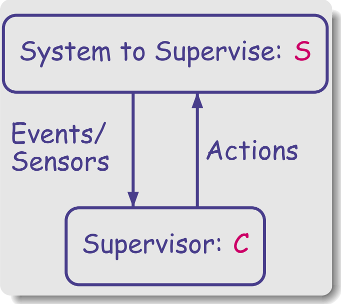

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Implementation
- Digital Supervisors
- Continuous Systems

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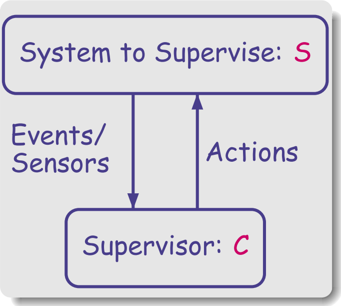

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Property φ

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- Digital Supervisors
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Outline of the Talk

- Control of Timed Systems: Basics
  - Verification and Control
  - Control $=$ Game
- Control of Discrete Event Systems
  - Games, Strategies, Winning States
  - Controllable Predecessors
  - Results for Finite Games
- Control of Timed Systems
  - Timed Automata
  - Timed Game Automata
  - Symbolic Algorithms for Timed Game Automata
- Advanced Subjects
  - Implementable Controllers
  - Optimal Controllers
  - Efficient Algorithms for Controller Synthesis
- Conclusion

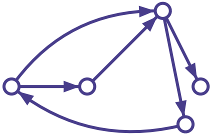
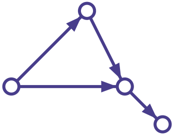
Next:

- Control of Timed Systems: Basics
  - Verification and Control
  - Control = Game
- Control of Discrete Event Systems
- Control of Timed Systems
- Advanced Subjects
- Conclusion

Verification and Control

Verification and Control

Modelling

 $\|$ 

Always (not bad)

$$S \;\|\; C \models \varphi$$

Verification and Control

Does the system meet the specification?

Modelling

Always (not bad)

$$S \parallel C \models \varphi$$

Verification and Control

Does the system meet the specification?

Modelling

Always (not bad)

$$S \quad \| \quad C \quad \models \quad \varphi$$

Model Checking Problem

Does the closed system $(S \parallel C)$ satisfy φ ?

Verification and **Control**

Can we enforce the system to meet the specification?

Modelling

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Modelling

$S \parallel C \models \varphi$

Control Problem

Can the open system S be restricted to satisfy φ ?
Is there a Controller C such that $(S \parallel C) \models \varphi$?

Verification and Control

Can we enforce the system to meet the specification?

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Always (not bad)

$S \parallel ?? \models \varphi$

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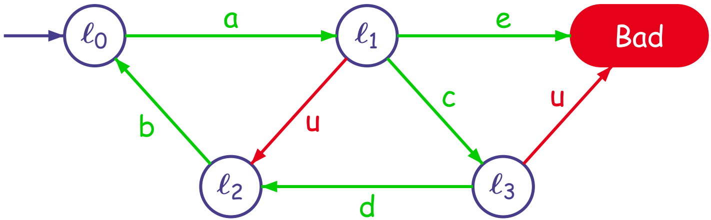
Control of Discrete Event Systems

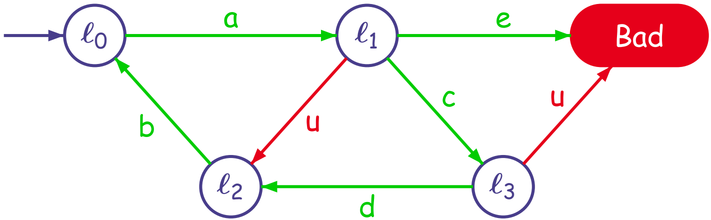


- Introduced by Ramadge & Wonham [Ramadge & Wonham'87]
- Discrete Event System = Finite Automaton with

> Controllable (Act_c) and Uncontrollable (Act_u) actions

- Specification = Control Objective = Language
  e.g. "avoid sequences of actions leading to state Bad"
- How to restrict: disable some controllable transitions
  [Ramadge & Wonham'89, Thistle & Wonham'94]

Control as Game



Open System = 2-player game, Controller (C) vs Environment (E)

- Controller plays Act_c moves, Environment plays Act_u moves
- Control Objective = Winning condition on the game
  "Avoid bad states" (safety) or "Enforce good states" (reachability)
- Control Problem: find a strategy (a controller) to win the game
- Various types of game models
  - Finite or pushdown or counter automata ...
  - Timed or hybrid automata

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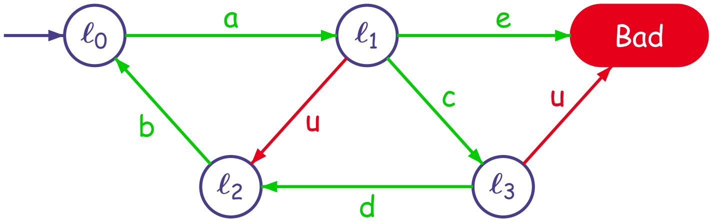

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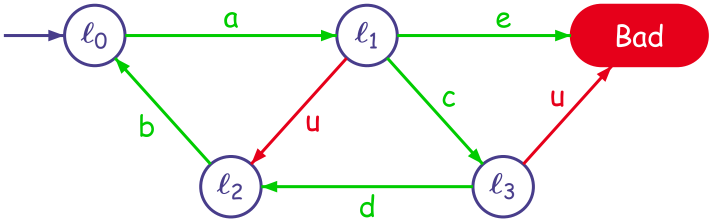

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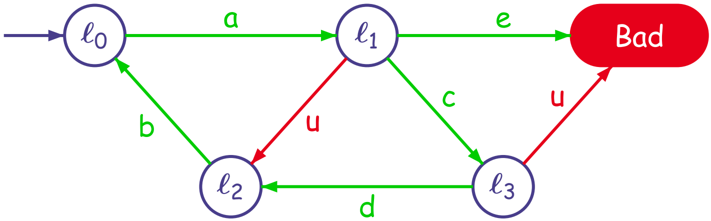

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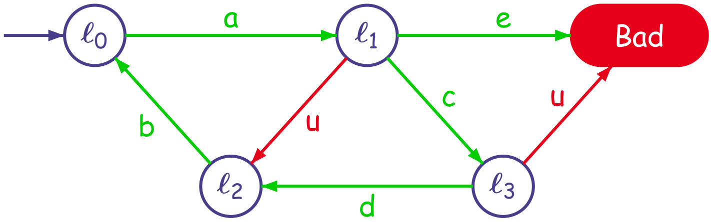

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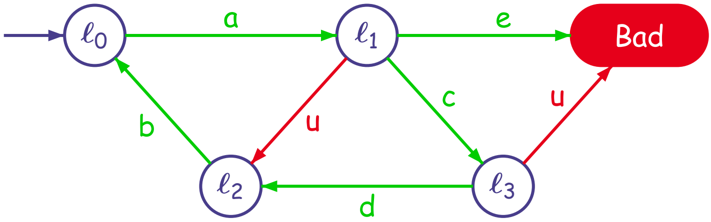

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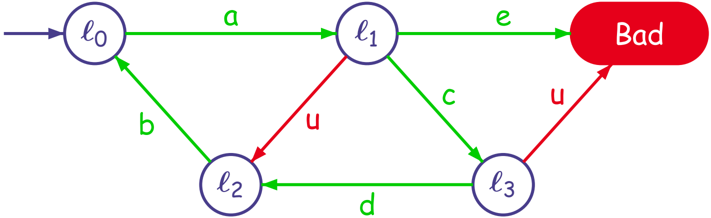

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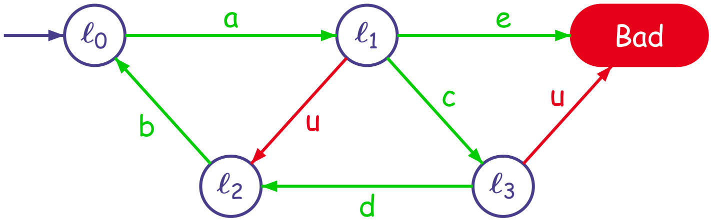

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Problems of Interest

Verification Problem (or Model Checking Problem)

Input: a model of the closed system S and a property φ
Problem: Does S satisfy φ ?

Control Problem $CP(G, \varphi)$

Input: a model of the open system (game) G and a property φ
Problem: Is there a strategy (controller) C s.t. $(C \parallel G)$ satisfy φ ?

Control Synthesis Problem (CSP)

Input: a model of the open system (game) G and a property φ
Problem: If the answer to the $CP(G, \varphi)$ is "yes", can we effectively compute a witness controller ?

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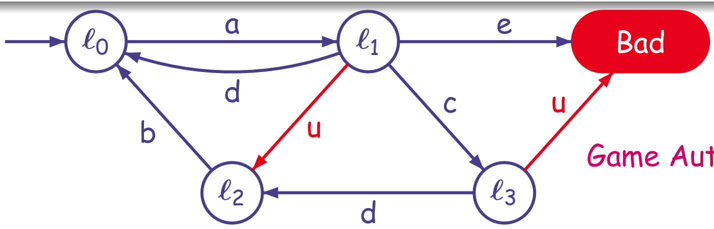
Game Automata, Strategies & Winning States



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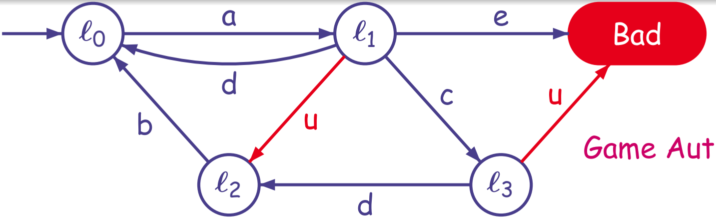


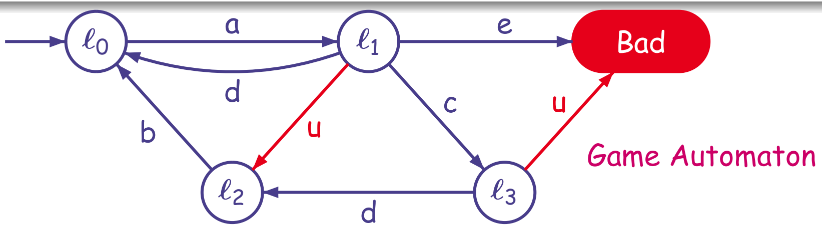
Runs

A run is a finite or infinite sequence of transitions $t_i = (s_i, \sigma_i, s_{i+1})$

$\ell_0 \xrightarrow{a} \ell_1 \xrightarrow{c} \ell_3 \xrightarrow{u} \text{Bad}$

$\ell_0 \xrightarrow{a} \ell_1 \xrightarrow{u} \ell_2 \xrightarrow{b} \ell_0 \xrightarrow{a} \ell_1 \xrightarrow{u} \ell_2 \xrightarrow{b} \quad \cdots$

Game Automata, Strategies & Winning States



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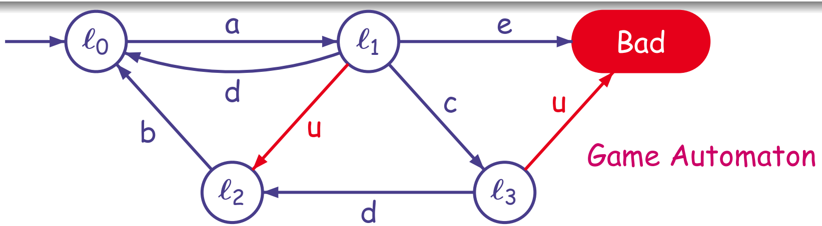

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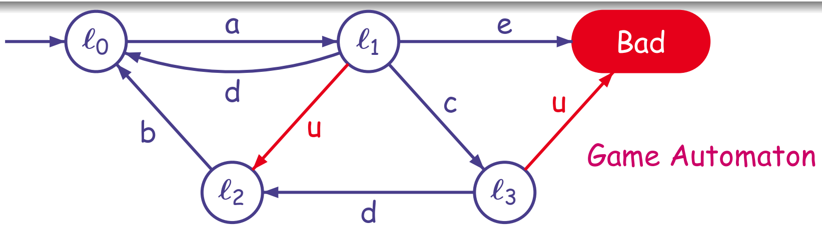
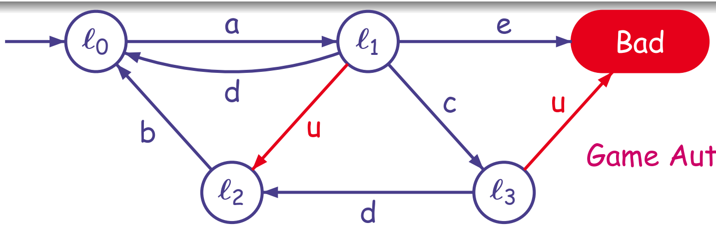


Strategy

- A strategy f gives for each finite run the controllable action to take. We assume full observability of the system

Game Automata, Strategies & Winning States



Game Automaton

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Example of Strategies

$f(\ell_0) = a$

$f(\ell_0 \xrightarrow{a} \ell_1) = c$

$f(\ell_0 \xrightarrow{a} \ell_1 \xrightarrow{u} \ell_2) = b$

$f(\ell_0 \xrightarrow{a} \ell_1 \xrightarrow{u} \ell_2 \xrightarrow{b} \ell_0 \xrightarrow{a} \ell_1) = e$

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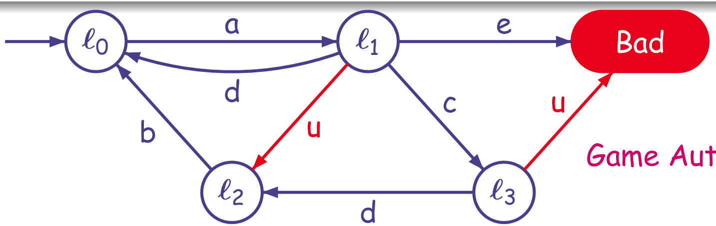

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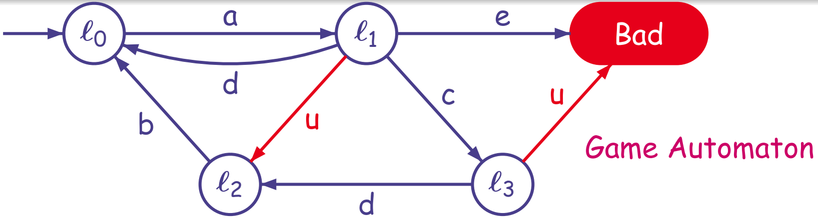

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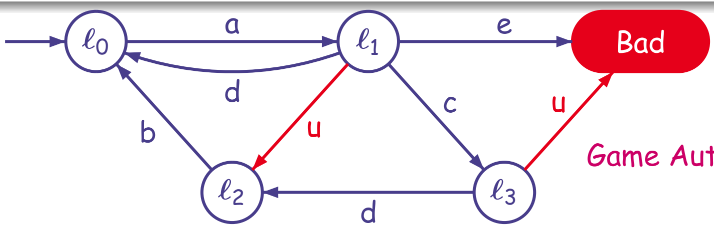

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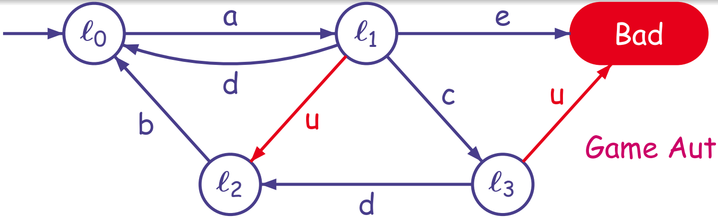

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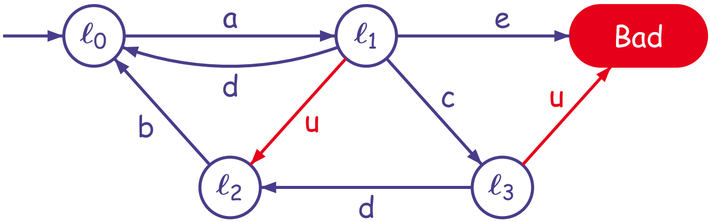
Winning States

A state s is winning if there exists a winning strategy from s.

Controllable Predecessors



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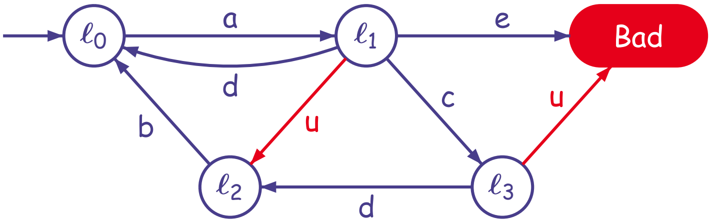
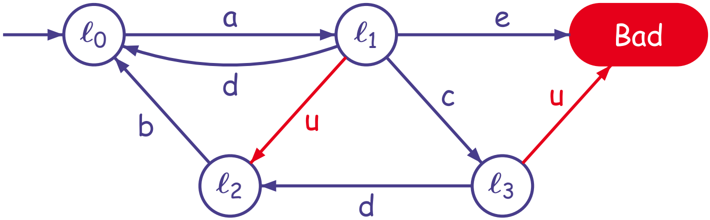


$\pi(X) =$ states from which one can enforce X with a controllable action

Controllable Predecessors



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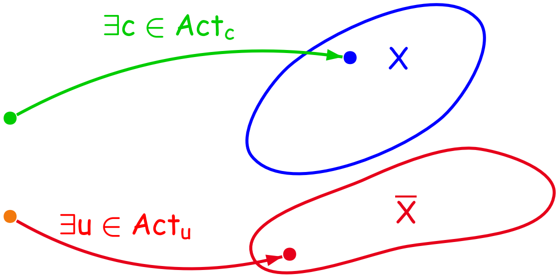

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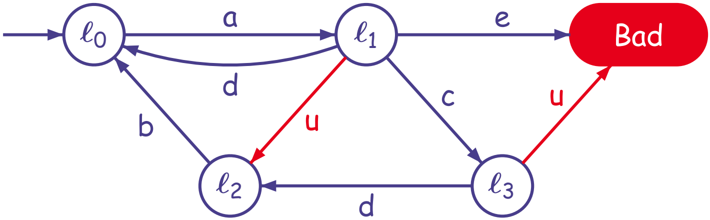

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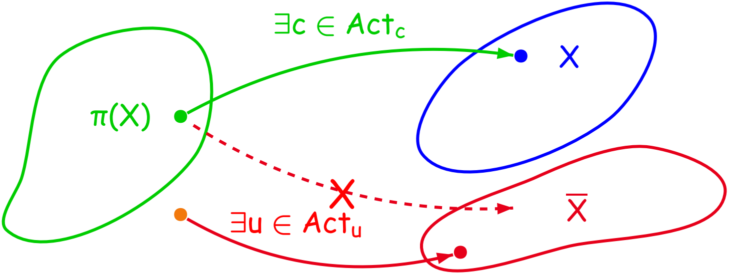

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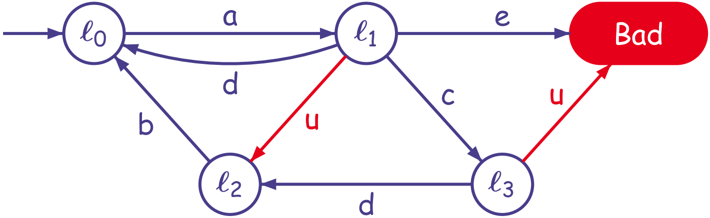

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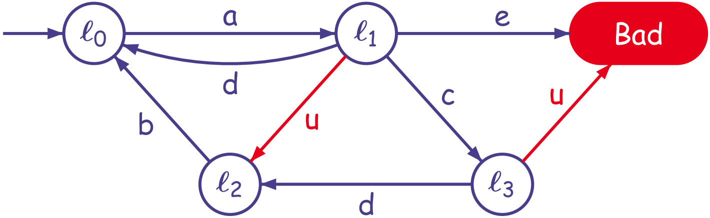
$$\pi(X) = \mathrm{Pred}^{Act_c}(X) \setminus \mathrm{Pred}^{Act_u}(\overline{X})$$

Fixpoint Characterization of Winning States for Safety Games:

1. Let φ be a set of safe (good) states and G a game
2. Let W^* be the greatest fixpoint of $h(X) = \varphi \cap \pi(X)$
3. W^* is the set of winning states for (G, φ)

Controllable Predecessors



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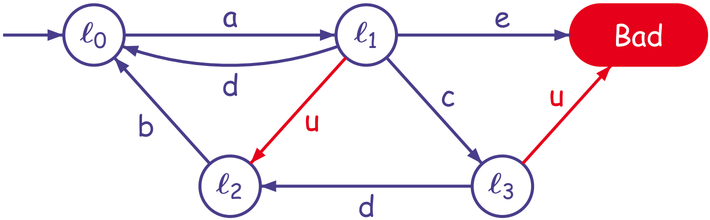

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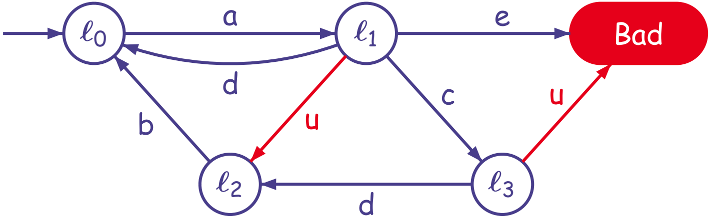

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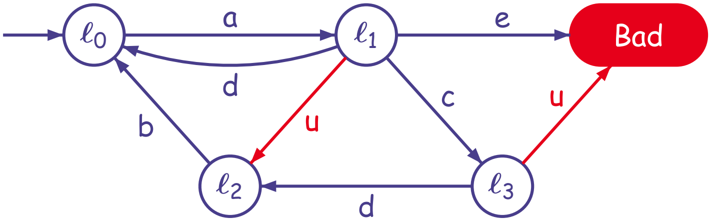

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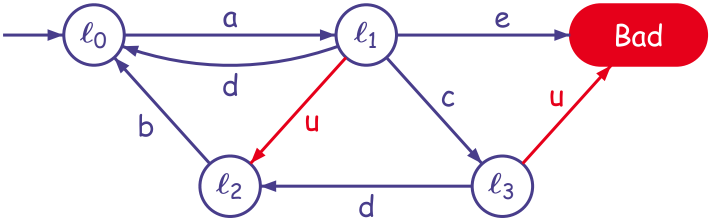

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- Decide CP: check that $\ell_0 \in W^*$
- Synthesis Problem: Given W^* and G, by def. of π we can build a winning strategy

Results for Finite Games

Given G a finite game, φ a control objective

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The fixpoint computation of W^* terminates

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Theorem (Positional Strategies are Sufficient)

Positional (or memoryless) strategies suffice to win finite-state (turn-based) games with ω-regular winning conditions.
(The number of states of C is $\leq$ number of states of G.)

Time !

- Context : Real-Time Critical Systems
- Some expected properties are quantitative properties
  e.g. scheduling
  or "The system will answer within 10 t.u. after a request is issued"
- One solution: discrete time
  - Can be "expensive"
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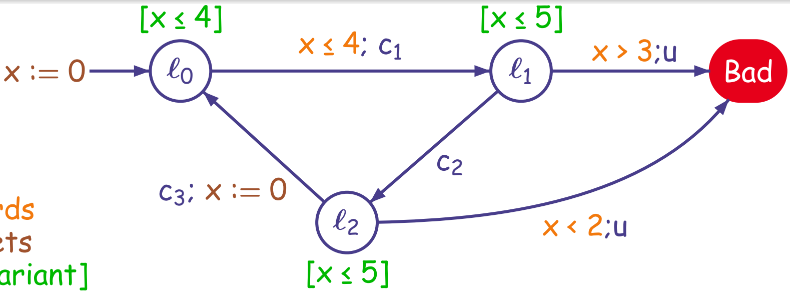
Next:

- Control of Timed Systems: Basics
- Control of Discrete Event Systems
- **Control of Timed Systems**
  - Timed Automata
  - Timed Game Automata
  - Symbolic Algorithms for Timed Game Automata
- Advanced Subjects
- Conclusion

Timed Automaton

[Alur & Dill'94]



Guards
Resets
[Invariant]

Timed Automaton = Finite Automaton + clock variables

Run = sequence of discrete and time steps

$\rho_1: \quad (\ell_0, 0) \xrightarrow{1.55} (\ell_0, 1.55) \xrightarrow{c_1} (\ell_1, 1.55) \xrightarrow{1.67} (\ell_1, 3.22) \xrightarrow{u} (\mathrm{Bad}, 3.22)$

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Zeno behaviour

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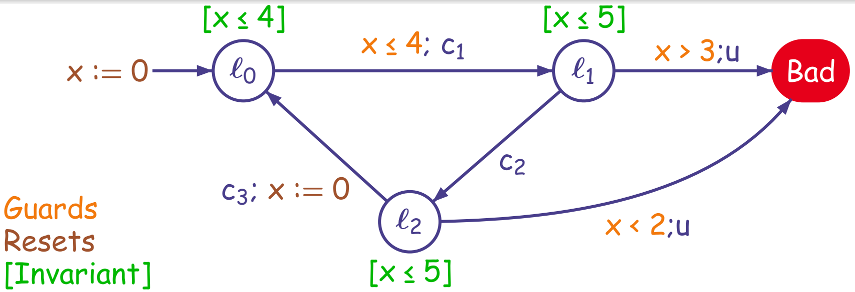

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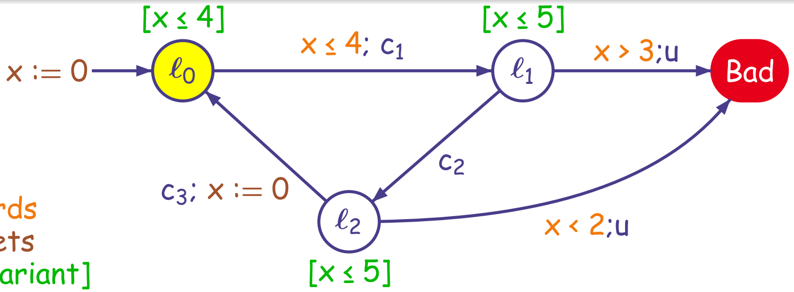

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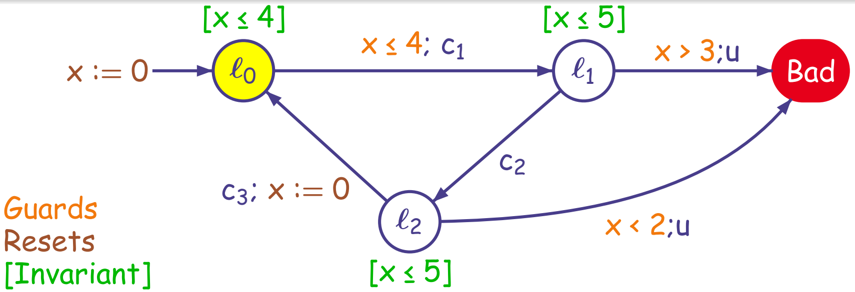

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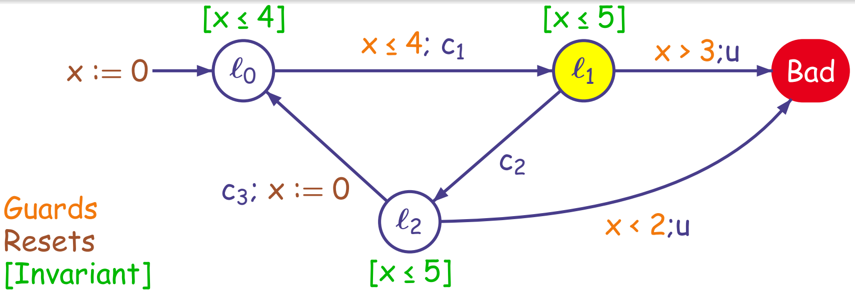

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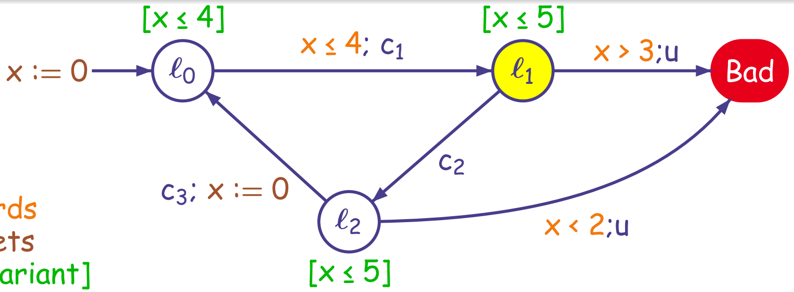

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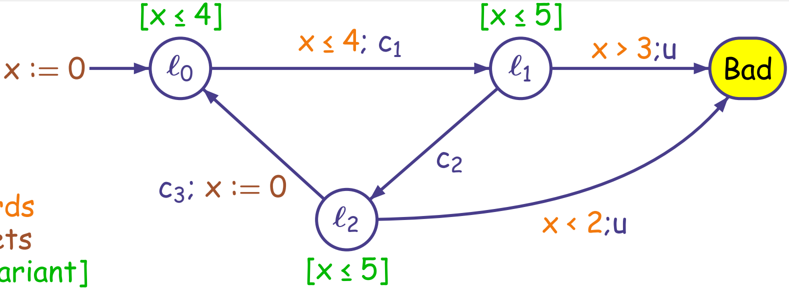

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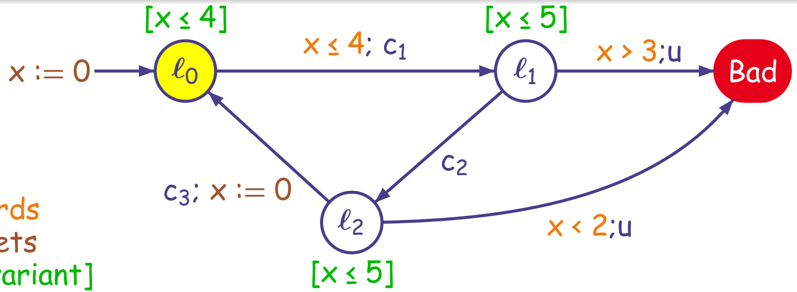

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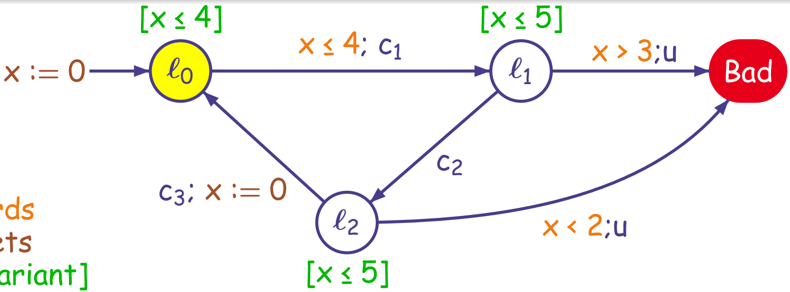

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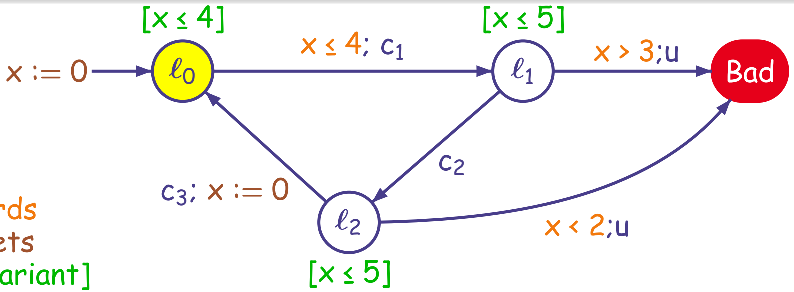

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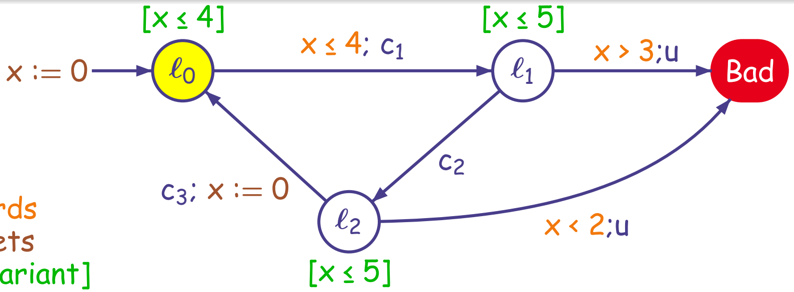

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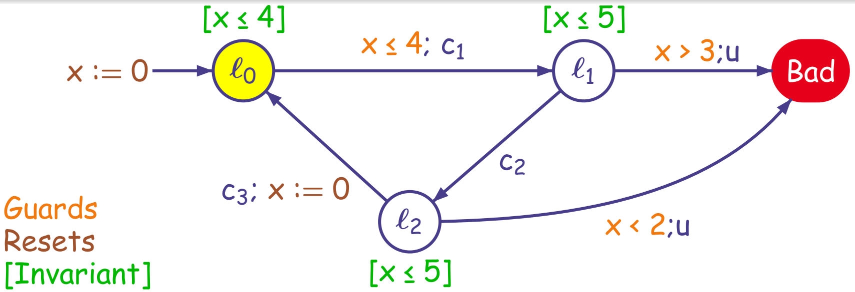

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States & Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ is the set of states of the TA $\qquad q = (\ell, v) \in Q$
- Discrete Successors of $X \subseteq Q$ by an action a:

$$Post^a(X) = \{q' \in Q \mid q \xrightarrow{a} q' \text{ and } q \in X\}$$

- Time Successors of $X \subseteq Q$:

$$Post^\delta(X) = \{q' \in Q \mid \exists t \geq 0 \mid q \xrightarrow{t} q' \text{ and } q \in X\}$$

- Zone = conjunction of triangular constraints
  $x - y < 3,\ x \geq 2 \wedge 1 < y - x < 2$
- Symbolic State is defined by a State predicate (SP)
  $P = \cup_{i \in [1..n]}(\ell_{j_i}, Z_i),\ \ell_{j_i} \in L,\ Z_i$ is a zone
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Effectiveness of $Post^a$ and $Post^\delta$

If P is a SP then $Post^a(P)$, $Post^\delta(P)$ are SP and can be computed effectively. (There is a symbolic version for $Post^a$ and $Post^\delta$.)

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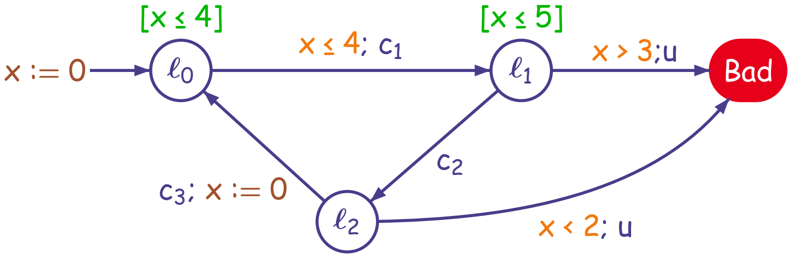
Decidability Result for TA

Region Graph

The Reachability Problem for TA is PSPACE-Complete.
Build a finite abstraction: region automaton

Timed Game Automata



- Introduced by Maler, Pnueli, Sifakis [Maler et al.'95]
- The controller continuously observes the system
  time elapsing and discrete moves are observable
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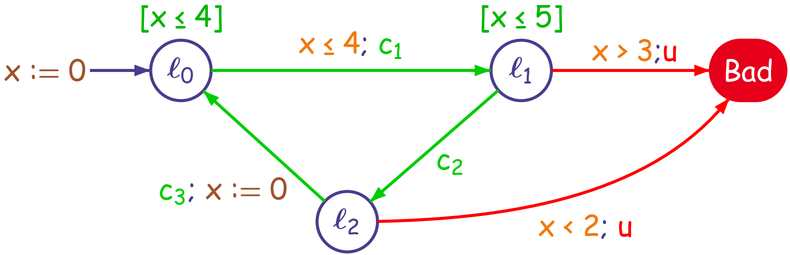
  - "do nothing" (delay action)
  - "do a controllable action" (among the ones that are possible)
- It can prevent time elapsing by taking a controllable move

Timed Game Automata



- Introduced by Maler, Pnueli, Sifakis [Maler et al.'95]
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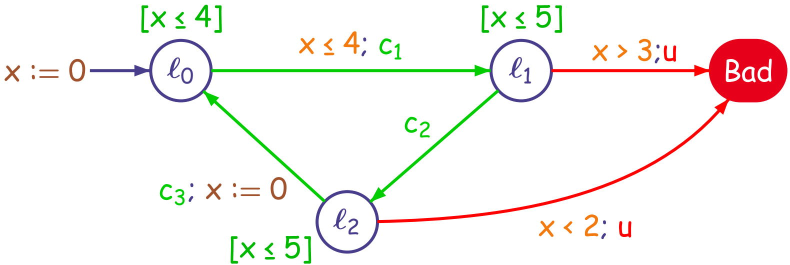
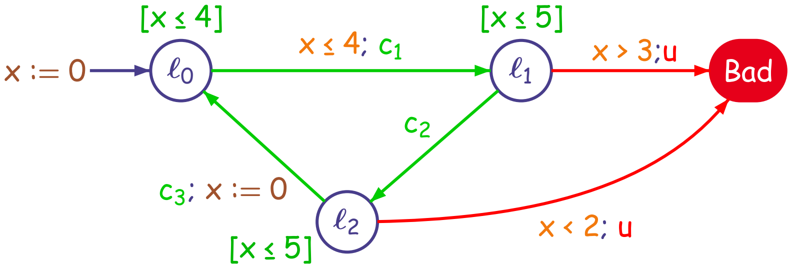
Strategies and Winning States



Strategies and Winning States



The strategy f: "Wait as long as the system permits"

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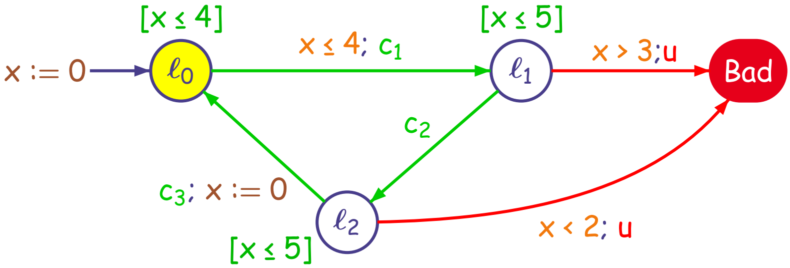
Strategies and Winning States



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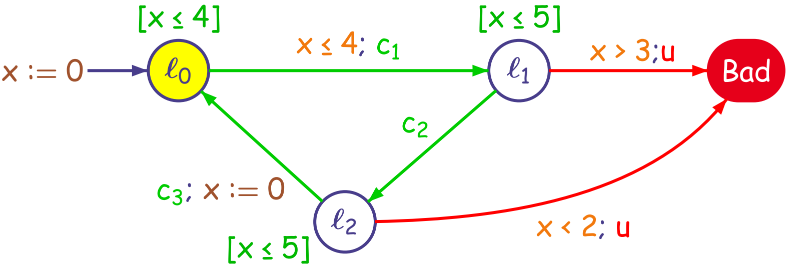

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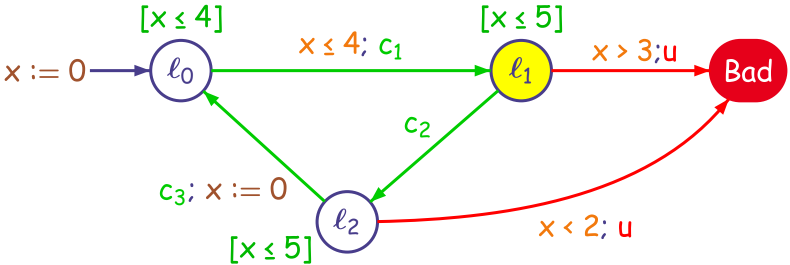

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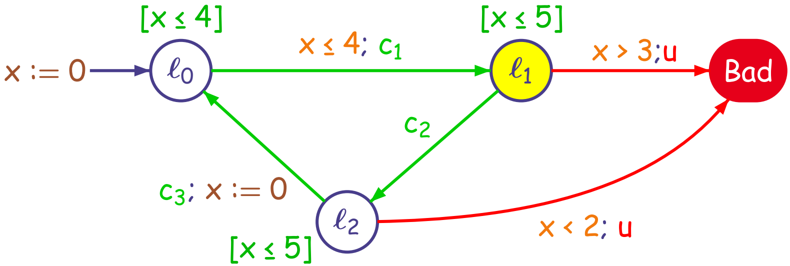

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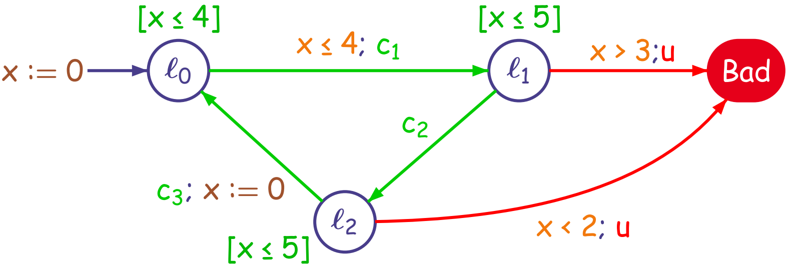

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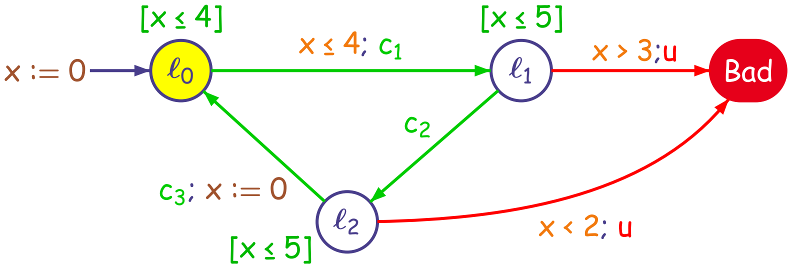
Strategies and Winning States



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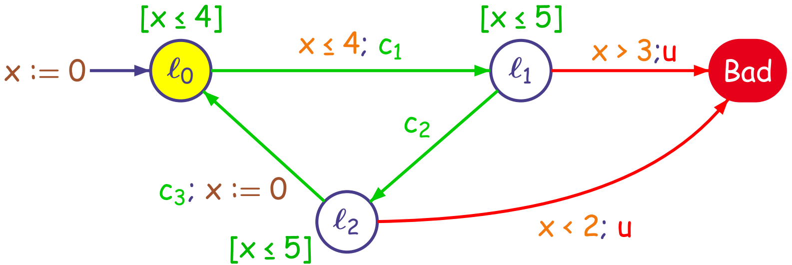

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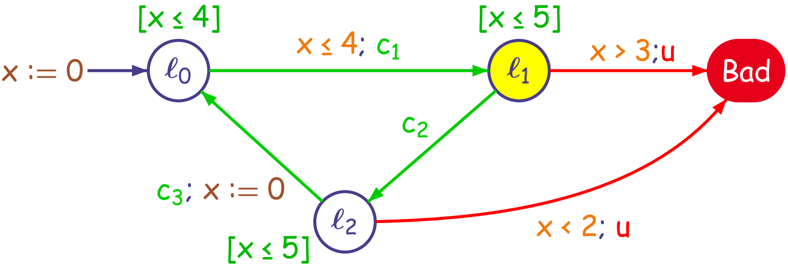

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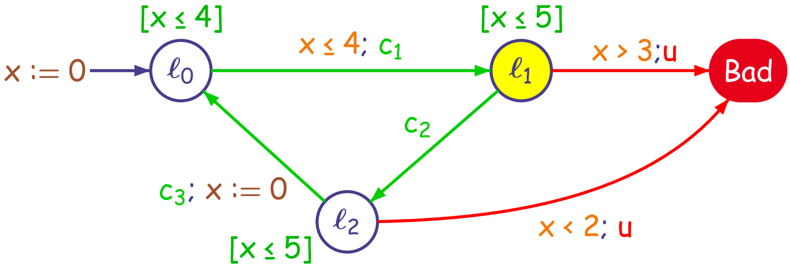
Strategies and Winning States



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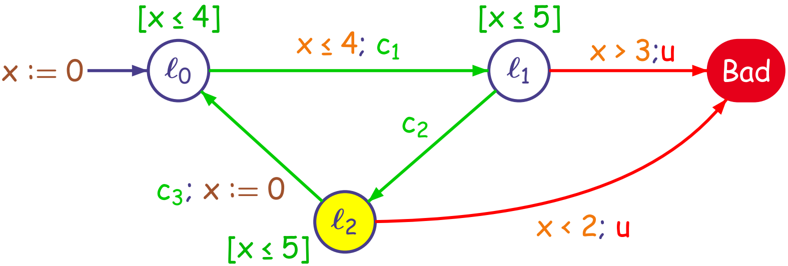
Strategies and Winning States



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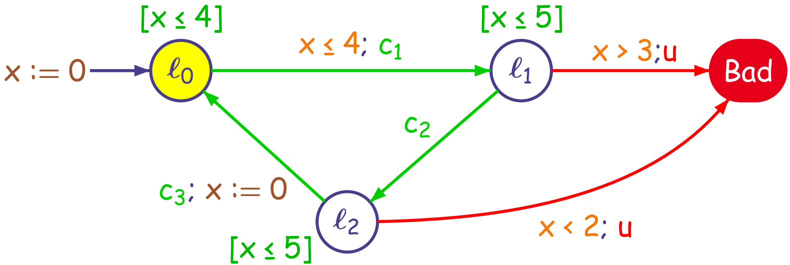

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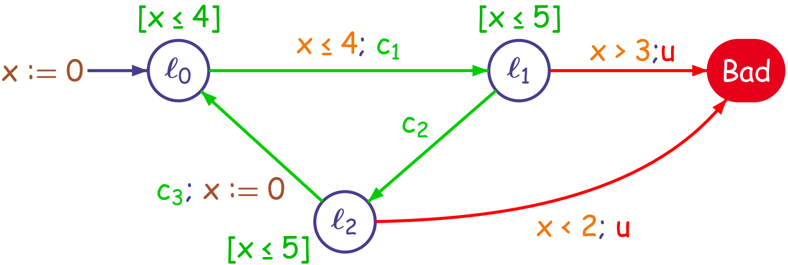

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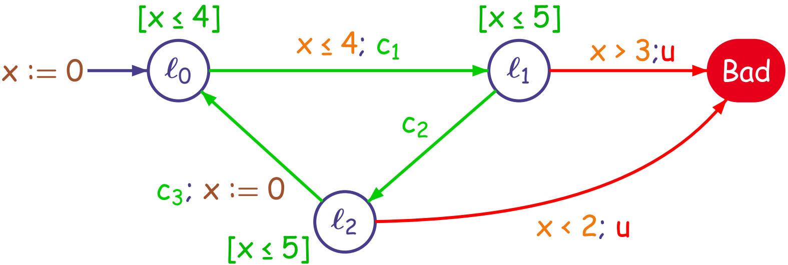

A winning strategy f'

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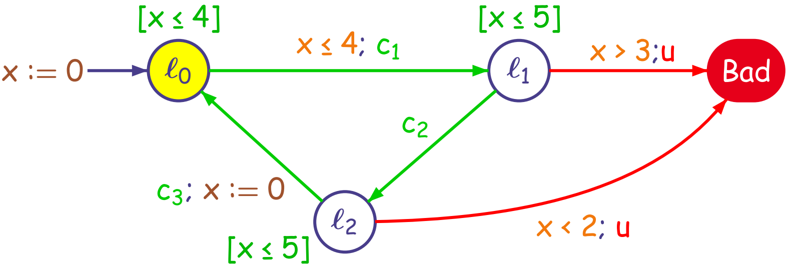

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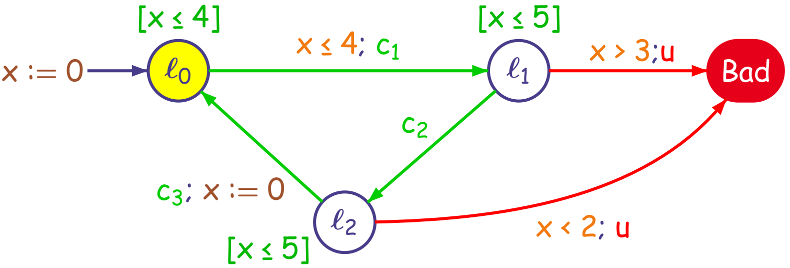

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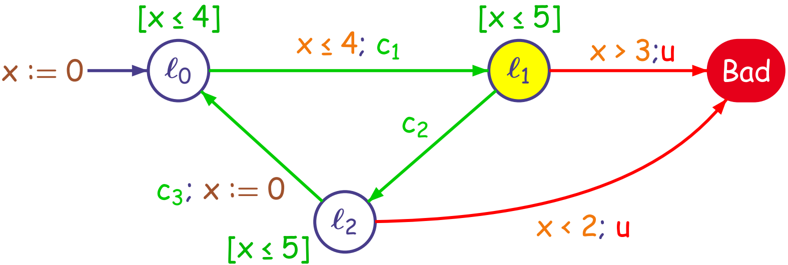

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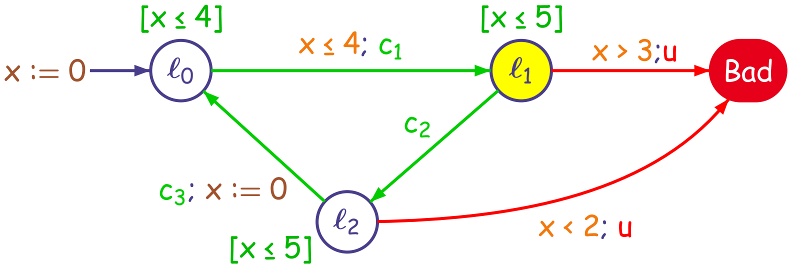

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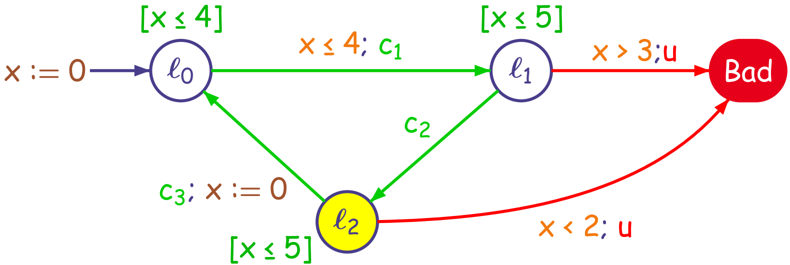
Strategies and Winning States



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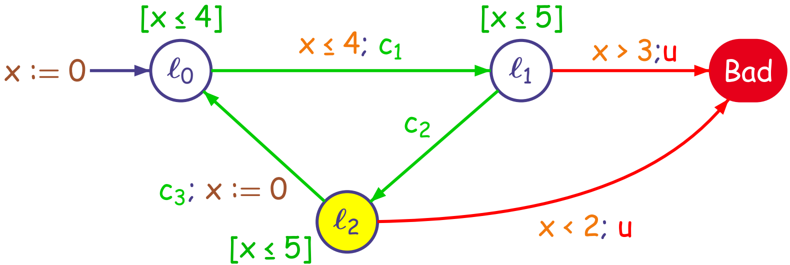

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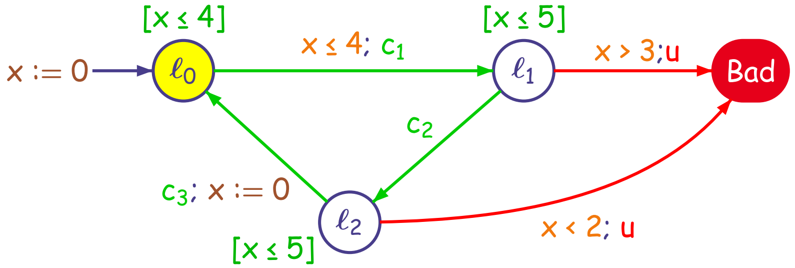
Strategies and Winning States



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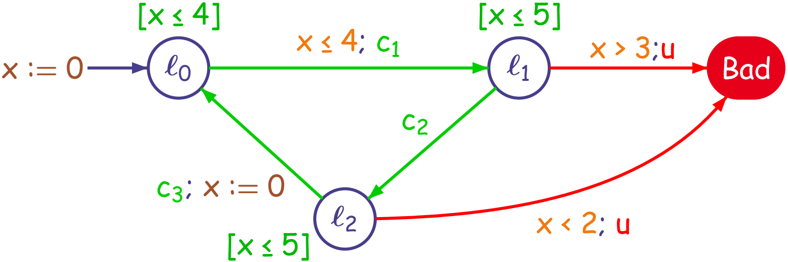
Strategies and Winning States



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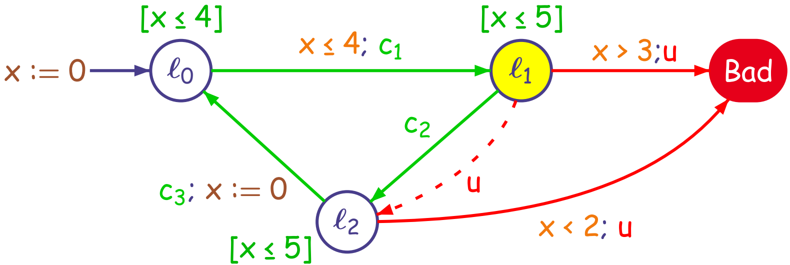
Strategies and Winning States



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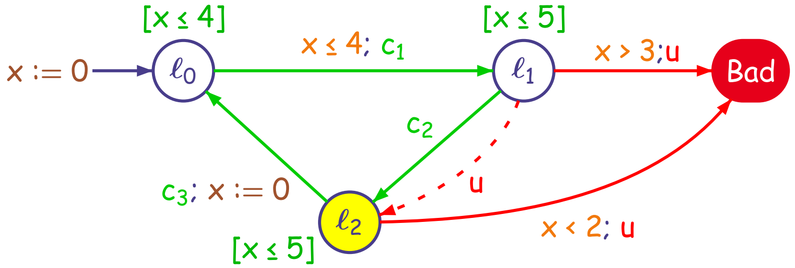

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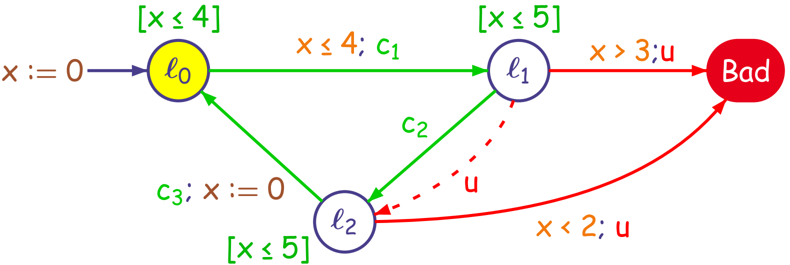
Strategies and Winning States



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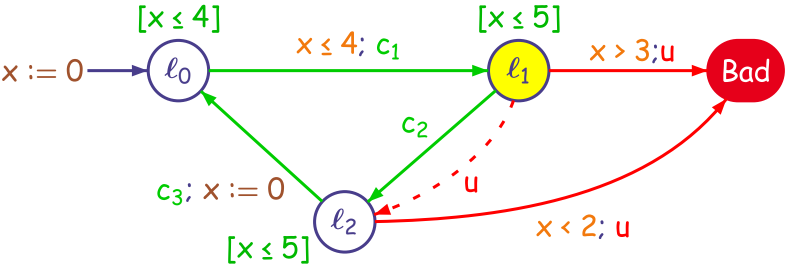
Strategies and Winning States



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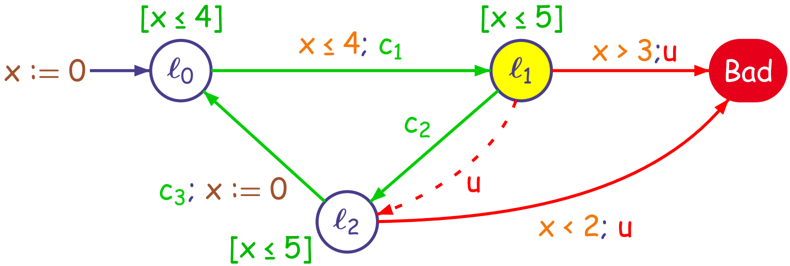

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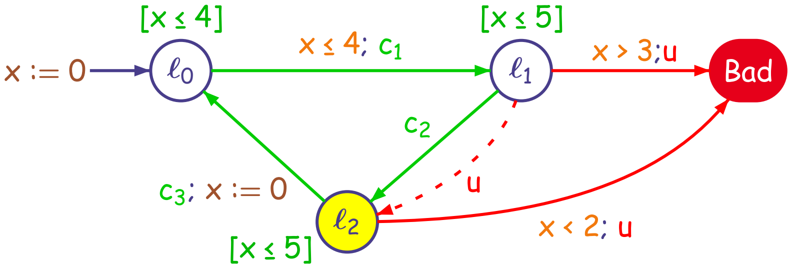

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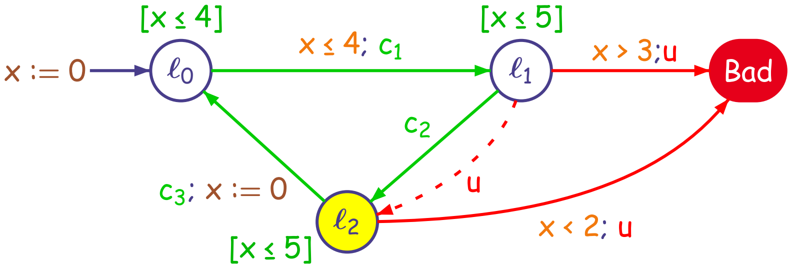
Strategies and Winning States



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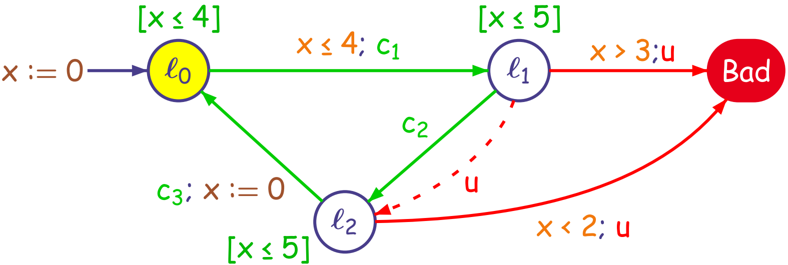
Strategies and Winning States



A winning strategy f'

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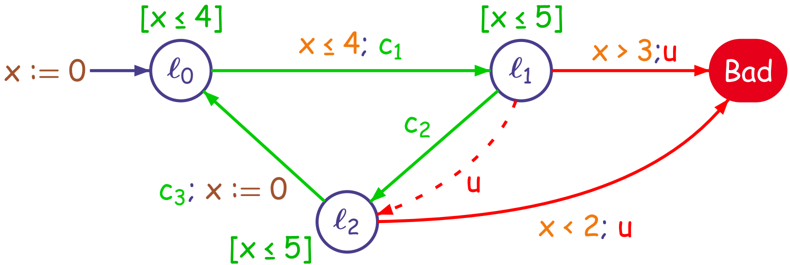
Strategies and Winning States



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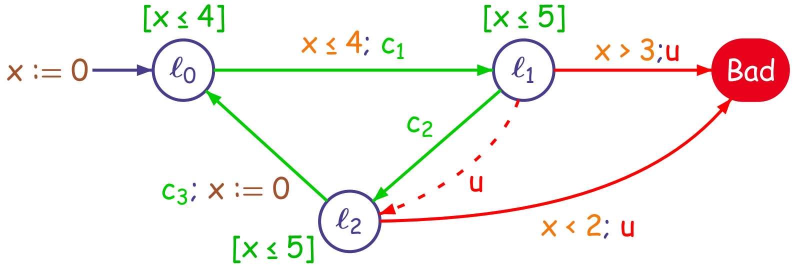
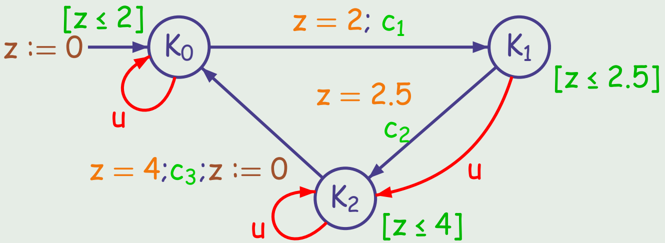
Strategies and Winning States



The Strategy f' as a Timed Automaton



Controllable Predecessors

$\pi(X, Y)$ = states from which one can enforce X and avoid Y by:
time elapsing followed by a controllable action

Fixpoint Characterization of Winning States for Safety Games:

1. Let φ be a set of safe (good) states and G a game
2. Let W^* be the greatest fixpoint of $h(X) = \varphi \cap \pi(X, \overline{X})$
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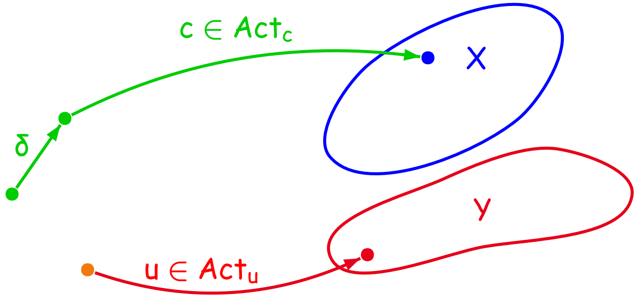

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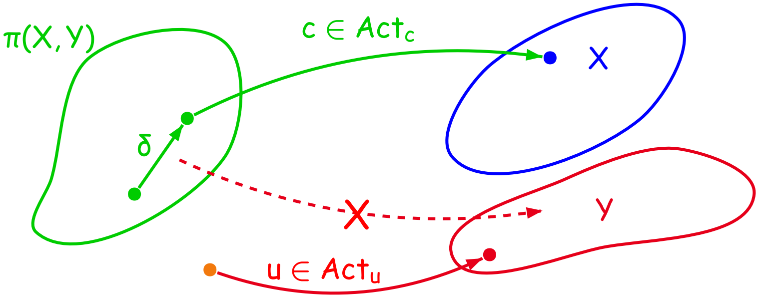

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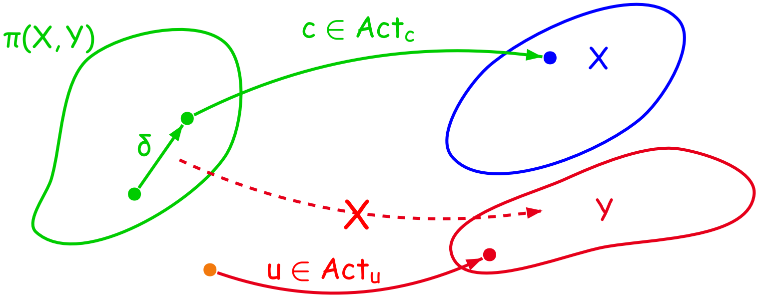

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Symbolic Algorithms for Safety Control

[Maler et al.'95, De Alfaro et al.'01]

Details & Example

1. There is a symbolic version for $\pi(X, Y)$
2. $\Longrightarrow$ there is a symbolic version for $h(X)$

Symbolic Algorithms for Safety Control

[Maler et al.'95, De Alfaro et al.'01]

Details & Example

1. There is a symbolic version for $\pi(X,Y)$
2. $\Longrightarrow$ there is a symbolic version for $h(X)$

- Control Problem (CP): check that $(\ell_0, 0) \in W^*$
- Control Synthesis Problem (CSP): by definition of π there is a strategy

Symbolic Algorithms for Safety Control

[Maler et al.'95, De Alfaro et al.'01]

▸ Details & Example

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Theorem (Termination)

The iterative computation of W^* terminates for (G, φ) with G a timed game automaton φ a ω-regular winning condition.

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The (Safety) Control Problem is decidable.

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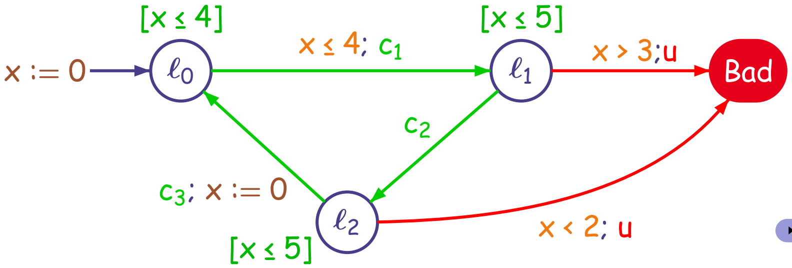
Theorem (Decidability of CP for Timed Game Automata)

The (Safety) Control Problem is decidable.

Theorem (Effectiveness of CSP)

If $(\ell_0, 0) \in W^*$ we can compute the most permissive positional winning strategy.

Result of the Computation for the Example



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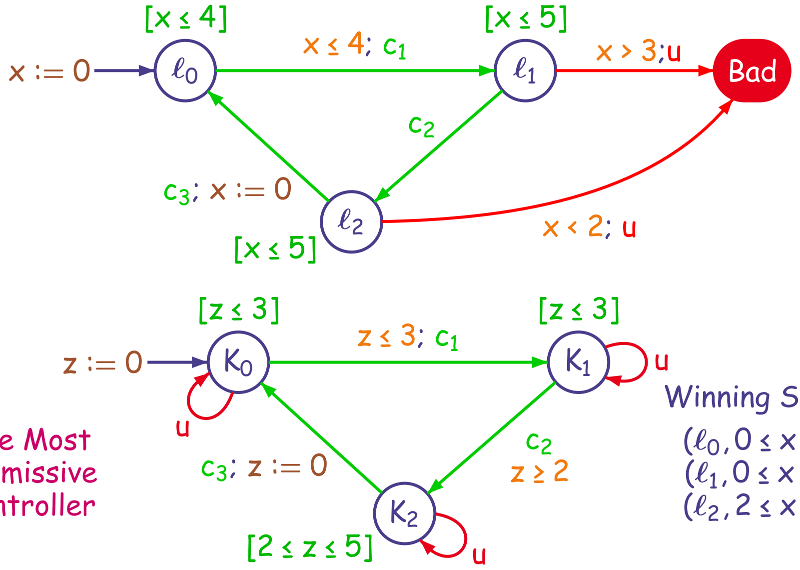


The Most Permissive Controller

Winning States

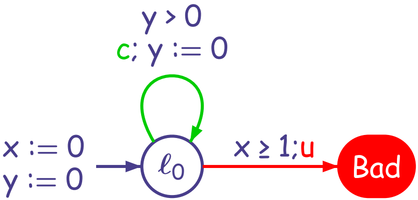
$(\ell_0, 0 \leq x \leq 3)$
$(\ell_1, 0 \leq x \leq 3)$
$(\ell_2, 2 \leq x \leq 5)$

Next:

- Control of Timed Systems: Basics
- Control of Discrete Event Systems
- Control of Timed Systems
- **Advanced Subjects**
  - Implementable Controllers
  - Optimal Controllers
  - Efficient Algorithms for Controller Synthesis
- Conclusion

Implementable Controllers

Problems with Dense-Time Control (1) [C. et al.'02]



The System

The Controller is Zeno !!!

Solution: add non-Zenoness in the control objective

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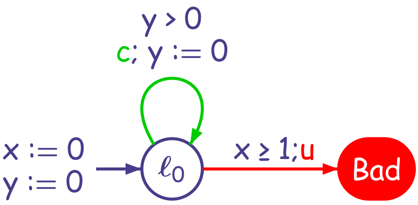

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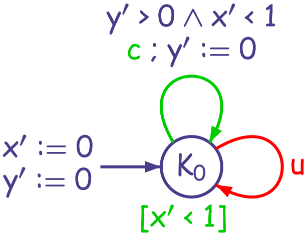

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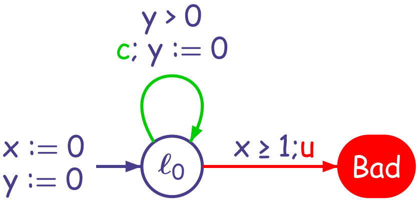

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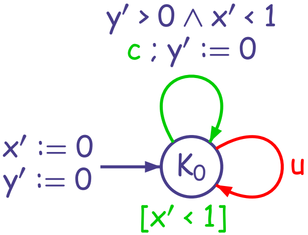

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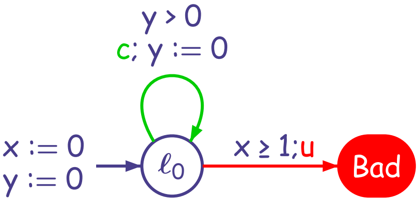

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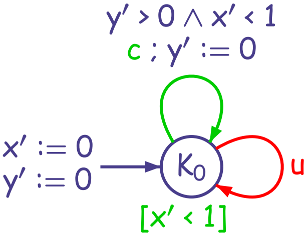

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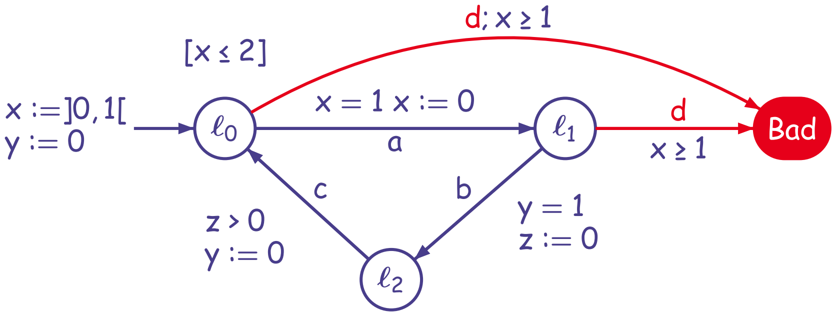
Problems with Dense-Time Control (2) [C. et al.'02]



- The controller is Non-Zeno; One untimed behavior: $(\ell_0\ell_1\ell_2)^\omega$
- Let $\Delta_k > 0$ be the time spent in ℓ_2 in the k-th loop from ℓ_0 to ℓ_0
- It implies: $\forall k, \sum_{i=1}^{k} \Delta_i < 1 - x_0$, with $\forall i, \Delta_i > 0$

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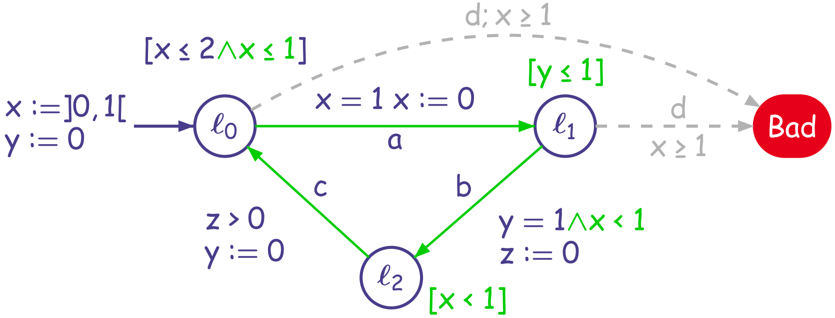

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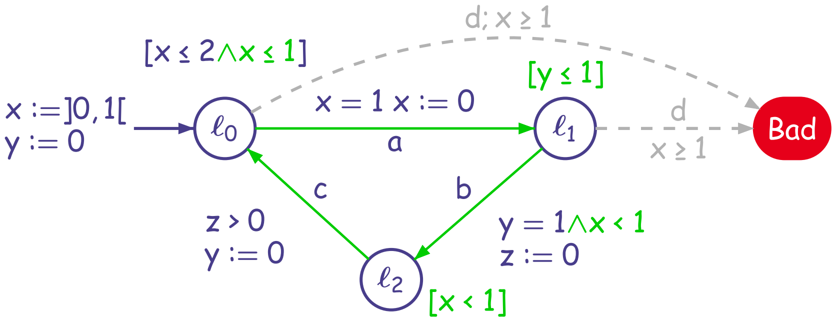

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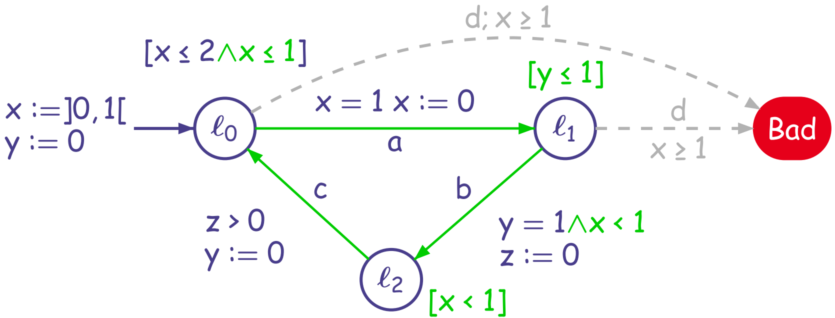

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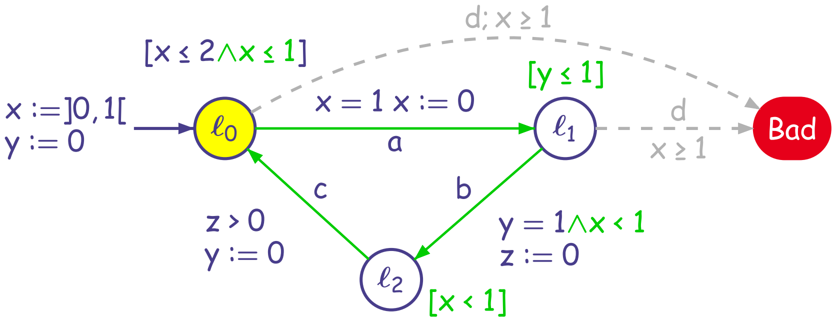

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	ℓ_0
x:	x_0
y:	0

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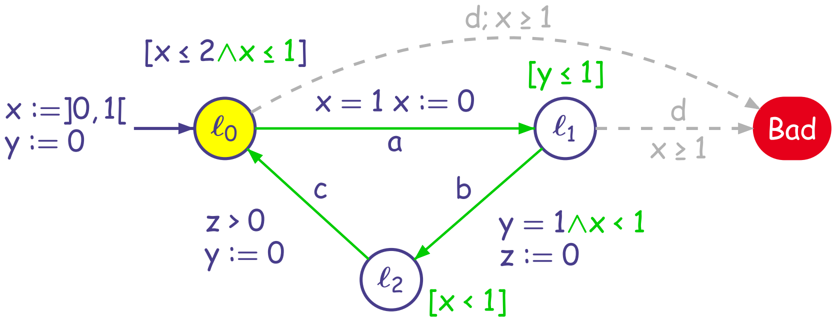

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	ℓ_0		
x:	x_0	$\rightsquigarrow$	1
y:	0		$1 - x_0$

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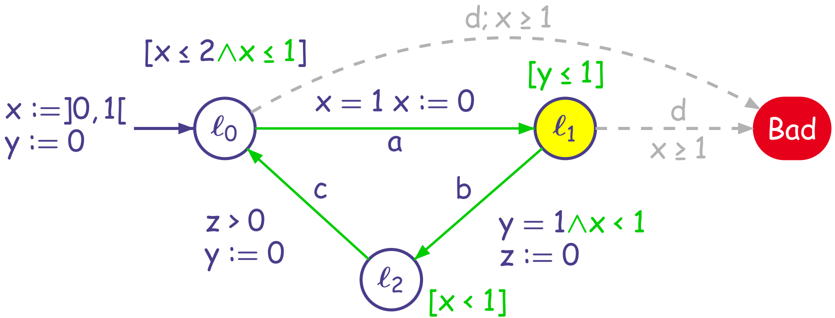

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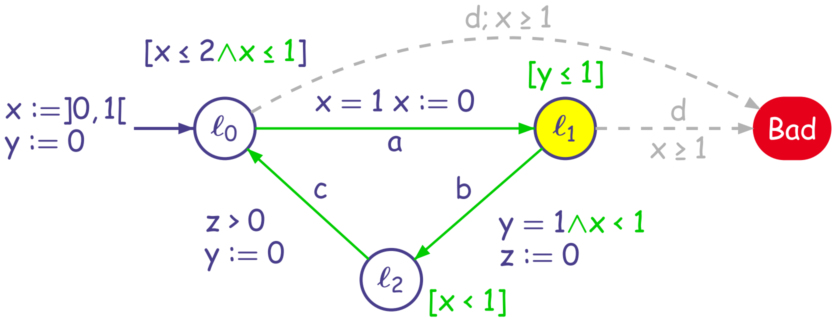
$$
\begin{array}{lcccc}
 & \ell_0 & & & \ell_1 \\
x: & x_0 & \rightsquigarrow & 1 & \xrightarrow{a} 0 \\
y: & 0 & & 1 - x_0 & 1 - x_0
\end{array}
$$

Problems with Dense-Time Control (2) [C. et al.'02]



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$$\begin{array}{llllll} & \ell_0 & & & \ell_1 & \\ x: & x_0 & \rightsquigarrow & 1 & \xrightarrow{a} & 0 & \rightsquigarrow & x_0 \\ y: & 0 & & 1 - x_0 & & 1 - x_0 & & 1 \end{array}$$

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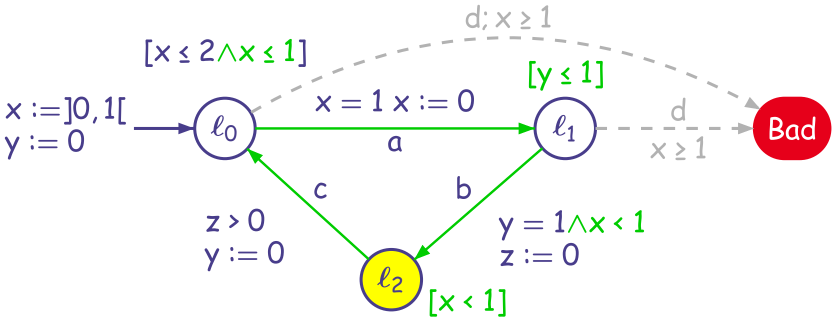

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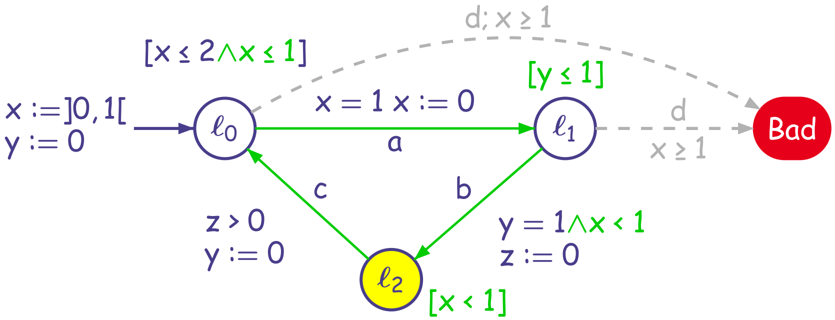

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$$
\begin{array}{lccccccccc}
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y: & 0 & & 1 - x_0 & & 1 - x_0 & & 1 & & 1 & & 1 + \Delta_1
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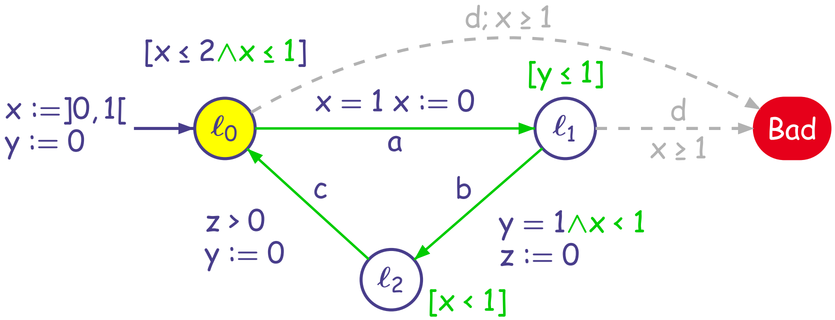

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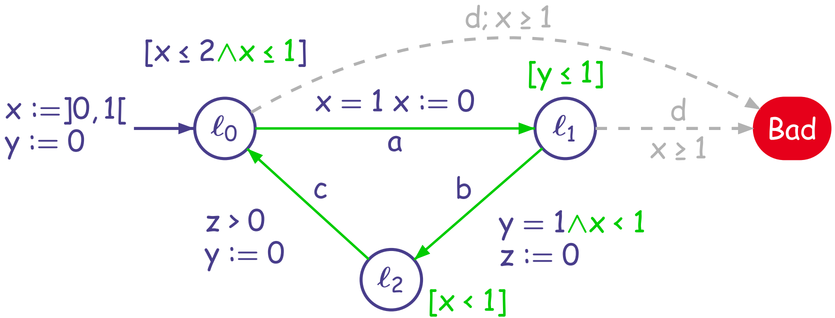
$$\begin{array}{lccccccccccc} & \ell_0 & & & & \ell_1 & & & & \ell_2 & & & & \ell_0 \\ x: & x_0 & \rightsquigarrow & 1 & \xrightarrow{a} & 0 & \rightsquigarrow & x_0 & \xrightarrow{b} & x_0 & \overset{\Delta_1}{\rightsquigarrow} & x_0 + \Delta_1 & \xrightarrow{c} & x_0 + \Delta_1 \\ y: & 0 & & 1 - x_0 & & 1 - x_0 & & 1 & & 1 & & 1 + \Delta_1 & & 0 \end{array}$$

Problems with Dense-Time Control (2) [C. et al.'02]



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$$\begin{array}{lcccccccccc}
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x: & x_0 & \rightsquigarrow & 1 & \xrightarrow{a} 0 & \rightsquigarrow & x_0 & \xrightarrow{b} & x_0 & \overset{\Delta_1}{\rightsquigarrow} & x_0+\Delta_1 & \xrightarrow{c} & x_0+\Delta_1 \\
y: & 0 & & 1-x_0 & 1-x_0 & & 1 & & 1 & & 1+\Delta_1 & & 0
\end{array}$$

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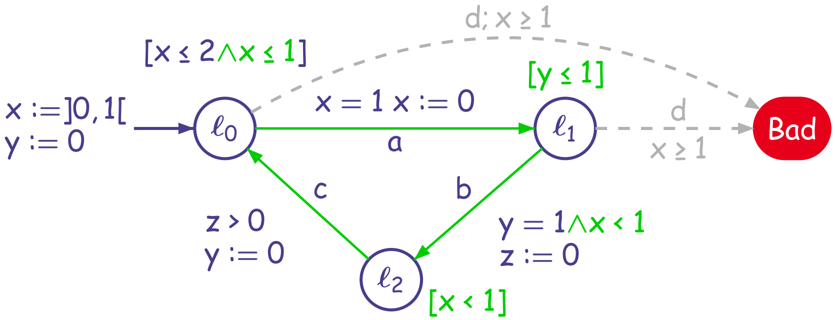

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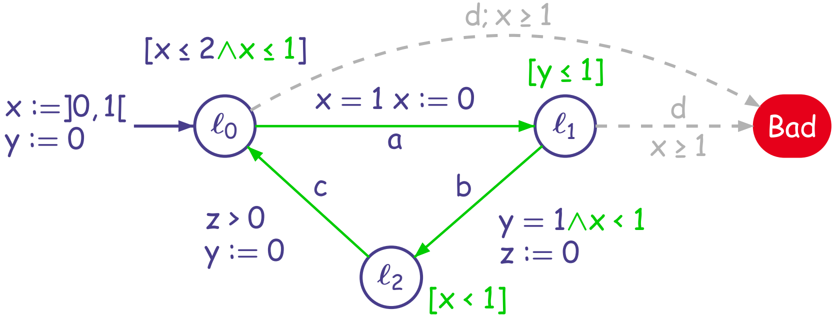

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The Controller is Non-Zeno but not Implementable !!!

Sampling Control

- Let $a \in \mathbb{Q}^*$ be a sampling rate
- An a-controller is a controller that can do actions only at $k \cdot a, k \geq 1$ and $k \in \mathbb{N}$

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Problem: Is there a a-controller for G that avoids Bad ?

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Input: $a \in \mathbb{Q}^*$, Bad (states), G a TGA
Problem: Is there a a-controller for G that avoids Bad ?

Theorem ([Henzinger & Kopke'99])

The Known Sampling Rate Control Problem is decidable.

Sampling Control

- Let $a \in \mathbb{Q}^*$ be a sampling rate
- An a-controller is a controller that can do actions only at $k \cdot a, k \geq 1$ and $k \in \mathbb{N}$

Unknown Sampling Rate Control Problem (USR)

Input: Bad (states), G a TGA
Problem: Is there a sampling rate $a \in \mathbb{Q}^*$ such that there is a a-controller for G that avoids Bad ?

Sampling Control

- Let $\alpha \in \mathbb{Q}^*$ be a sampling rate
- An α-controller is a controller that can do actions only at $k \cdot \alpha, k \geq 1$ and $k \in \mathbb{N}$

Unknown Sampling Rate Control Problem (USR)

Input: Bad (states), G a TGA
Problem: Is there a sampling rate $\alpha \in \mathbb{Q}^*$ such that there is a α-controller for G that avoids Bad ?

Theorem ([C. et al.'02])

The Unknown Sampling Rate Control Problem is undecidable.

Summary of the Results

Decidability results for the safety control problem on LHA:

	Known Switch Cond.	Unknown Switch Cond.
Timed Auto.	√ [Maler et al.'95]	√ [Maler et al.'95]
Init. Rect. Auto	√ [Henzinger et al.'99]	× [Henzinger et al.'95]
Rect. Auto.	× [Henzinger et al.'99]	× [Henzinger et al.'99]

	Known Sampling Rate	Unknown SR
Timed Auto.	√ [Hoffmann & Wong-Toi'92]	× [C. et al.'02]
Init. Rect. Auto.	√ [Henzinger & Kopke'97]	× [C. et al.'02]
Rect. Auto.	√ [Henzinger & Kopke'97]	× [C. et al.'02]

√: Decidable ×: Undecidable

Recent result [Bouyer et al.'06]

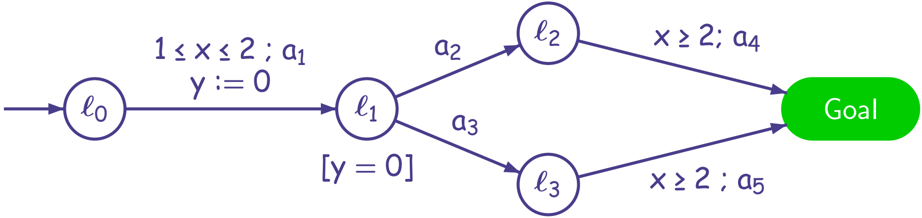
The reachability USR-CP is decidable for o-minimal automata.

Results on implementation of Timed Automata

[De Wulf et al.'04b, De Wulf et al.'04a, De Wulf et al.'05b]

Optimal Controllers

Optimal Reachability for Timed Automata



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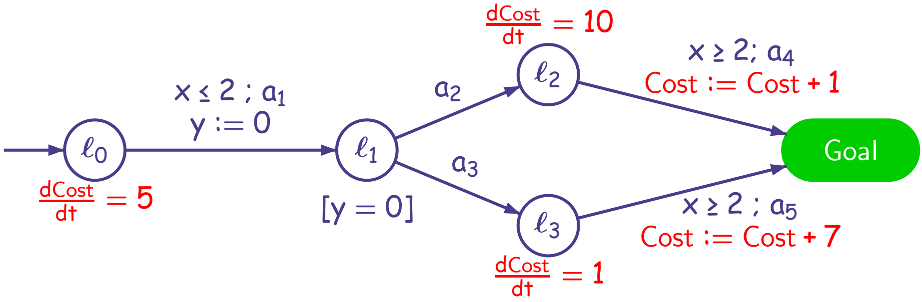
Optimal Reachability for Timed Automata



- Reachability for Timed Automata [Alur & Dill'94]
- Optimal Reachability for Priced (or Weighted) Timed Automata [Larsen et al.'01, Alur et al.'01]

$$(\ell_0,0,0) \xrightarrow{1} (\ell_0,1,1) \xrightarrow{a_1\ a_2} (\ell_2,1,0) \xrightarrow{3} (\ell_2,4,3) \xrightarrow{a_4} (Goal,4,3)$$

$$Cost = 1 \cdot 5 + 3 \cdot 10 + 1 = 36$$

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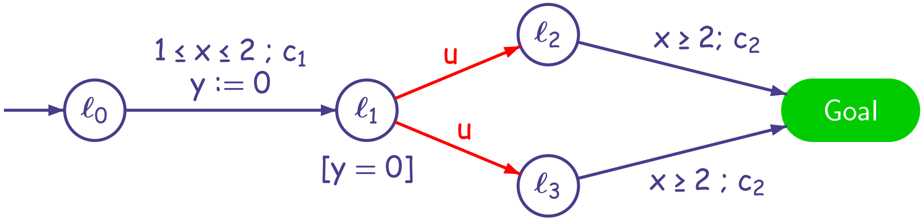

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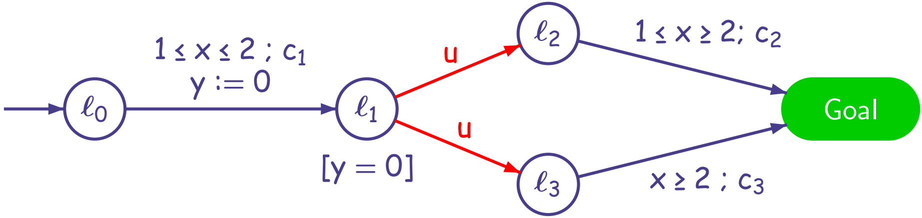

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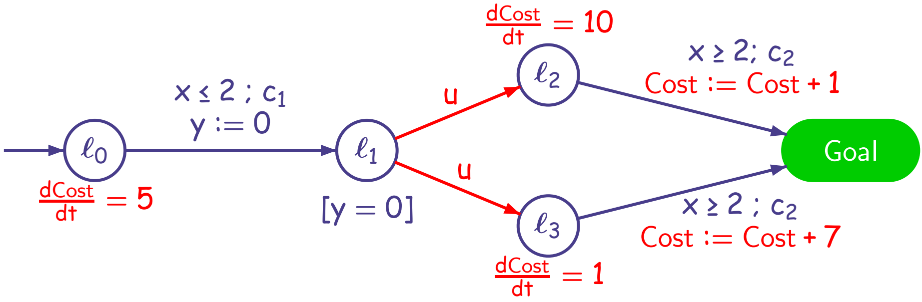

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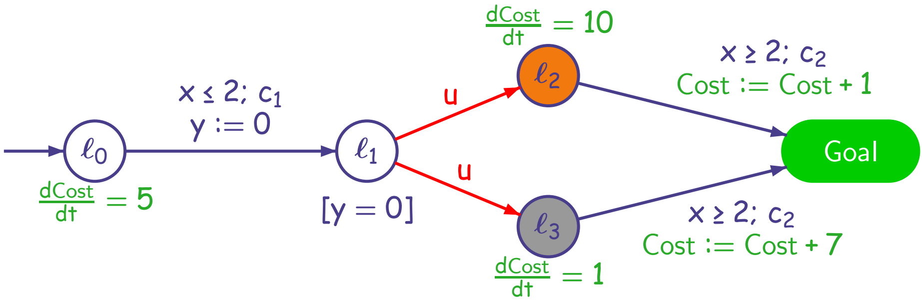
Optimal Control for Priced Timed Game Automata ?

A Small Example



- What is the best cost whatever the environment does ?

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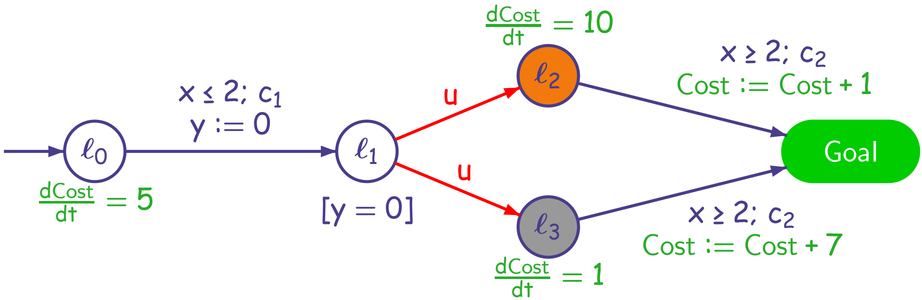

- What is the best cost whatever the environment does ?

$$\inf_{0 \leq t \leq 2} \max\{5t + 10(2-t) + 1, 5t + (2-t) + 7\} = 14 + \frac{1}{3}$$

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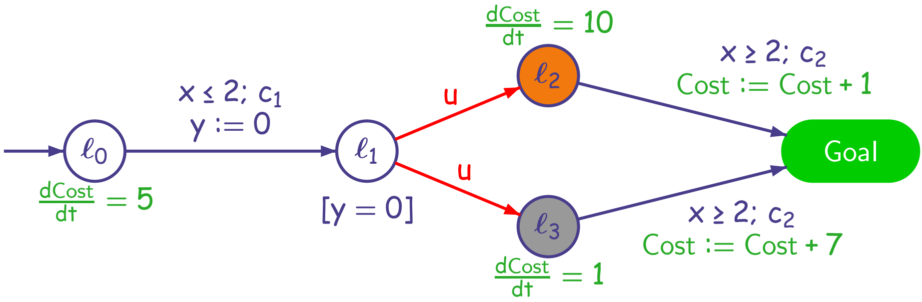

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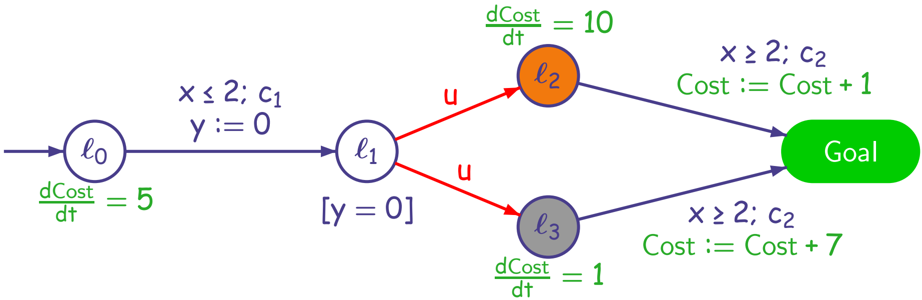

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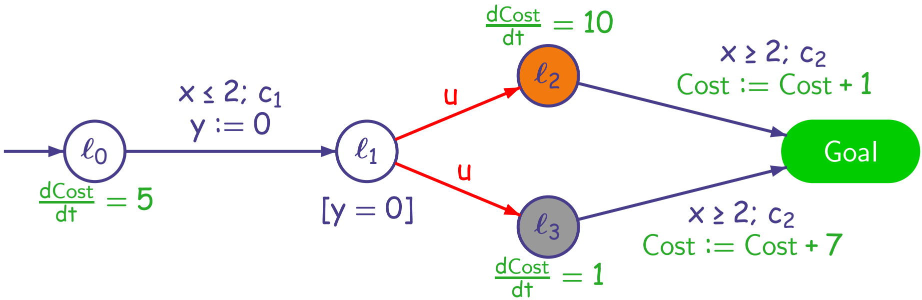

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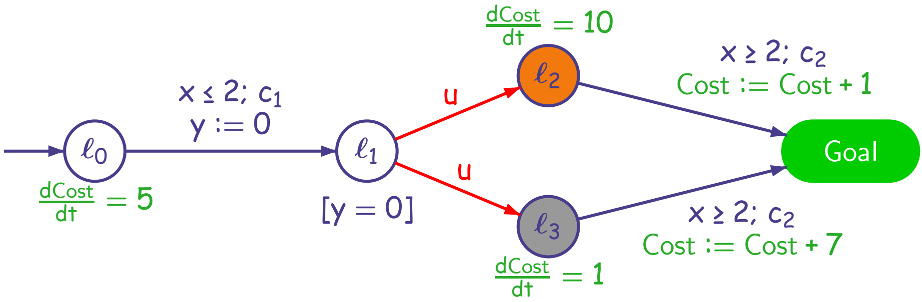

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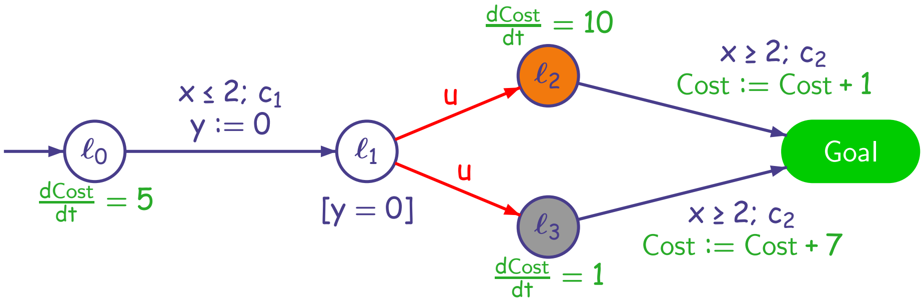

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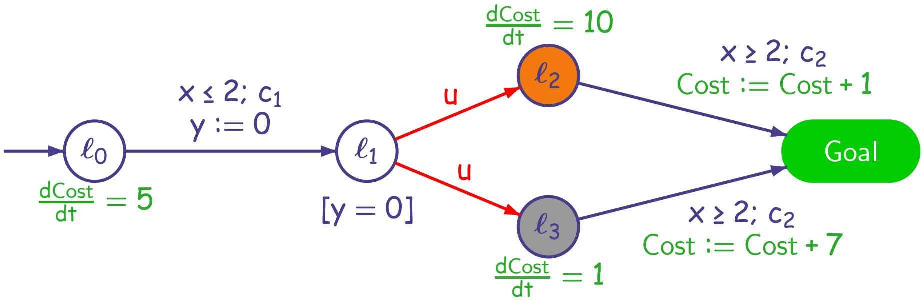

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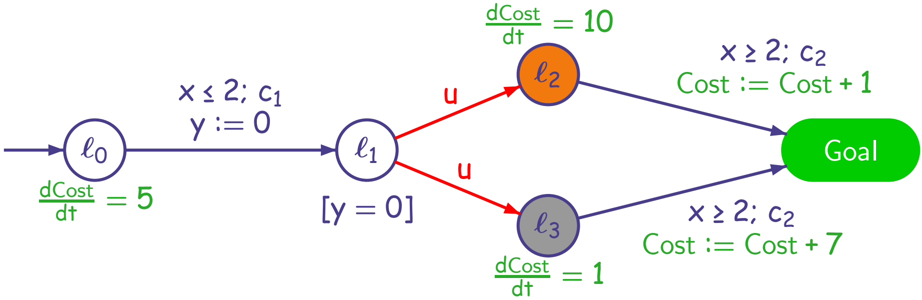
  Yes: wait in ℓ_0 until $t = \frac{4}{3}$ and then fire c_1
- Can we compute such a strategy ?

  Yes: but need memory sometimes

Optimal Control Problems

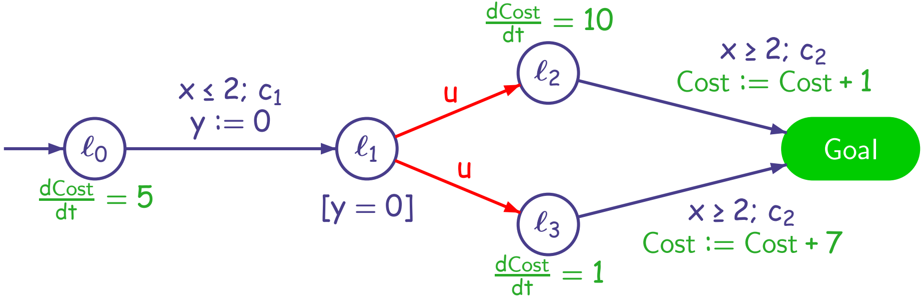


Can we find algorithms for these problems on PTGA ?

1. Compute the optimal cost
2. Decide if there is an optimal strategy
3. Compute an optimal strategy (if one exists)

From Optimal Control to Control

A Reachability TGA $\mathcal{A}$

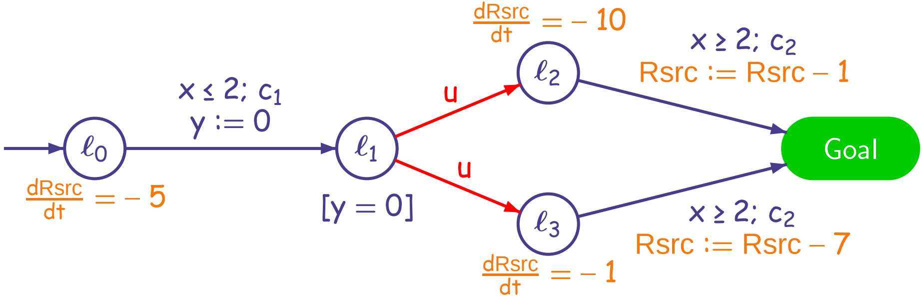


- Transform $\mathcal{A}$ in Linear Hybrid Game Automaton $H(\mathcal{A})$
- Define the reachability game for $H(\mathcal{A})$ with goal: $\text{Goal} \wedge \text{Rsrc} \geq 0$

Optimal Control for $\mathcal{A}$ $\iff$ Reachability Control for $H(\mathcal{A})$

From Optimal Control to Control

A Linear Hybrid Game $H(\mathcal{A})$



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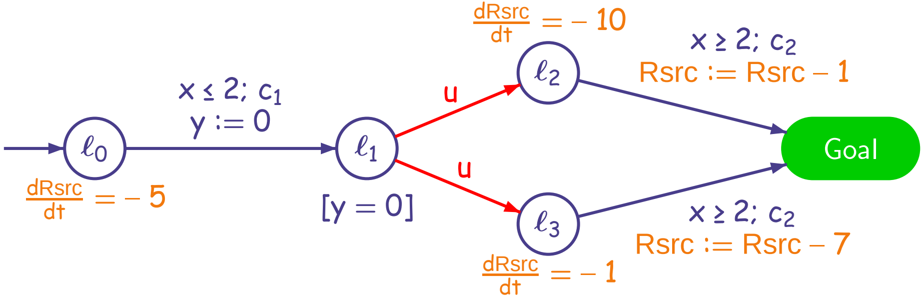

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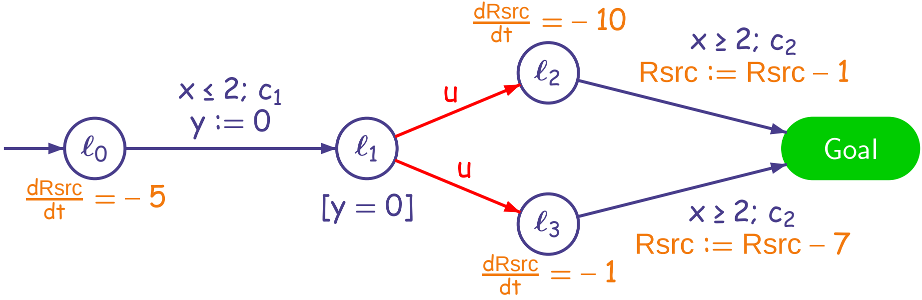

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Results [Bouyer et al.'04a, Bouyer et al.'04b]

Theorem (Reachability Control for LHA)

There is a semi-algorithm CompWin that computes the set of winning states for LHA.
Uses polyhedra instead of zones.

Results

[Bouyer et al.'04a, Bouyer et al.'04b]

Let $\mathcal{A}$ be a Reachability Priced Timed Game Automaton such that:

- $\mathcal{A}$ is cost non-zeno *i.e.* $\exists\kappa$ s.t. every cycle in the region automaton of $\mathcal{A}$ has cost at least κ
- $\mathcal{A}$ is bounded *i.e.* all clocks in $\mathcal{A}$ are bounded

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Theorem ([Brihaye et al.'05])

Non-Zeno Cost is a necessary assumption.

Summary of the Results

What's decidable about optimal reachability control?

- Non-Zeno Cost [Bouyer et al.'04a]
- O-minimal automata [Bouyer et al.'07]
- 1-clock PTGA (3EXPTIME) [Bouyer et al.'06a]

What's UNdecidable about optimal control?

- 5-clock Zeno PTGA [Brihaye et al.'05]
- 3-clock Zeno PTGA [Bouyer et al.'06b]

What's decidable for infinite schedules (safety) ?

- Mean Cost decidable for 1-player PTA [Bouyer et al.'04c]

What's open?

Optimal Mean Cost for PTGA

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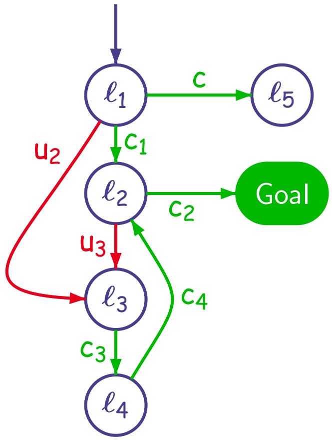
Efficient Controller Synthesis

Reachability Control for Finite Games

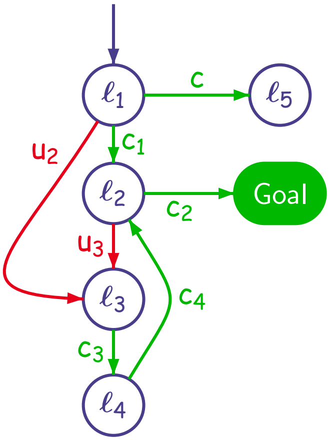


$\longrightarrow$ Uncontrollable
$\longrightarrow$ Controllable

Aim: enforce Goal

- Semantics: no priority
  Cont. must take a controllable action
- Winning run = a run containing Goal
- Strategy: based on the full history tells which controllable action to fire
  It restricts the set of behaviors of the open system
- Winning strategy: all the runs in the controlled system are winning
- Winning state = a state from which there is winning strategy

Reachability Control for Finite Games



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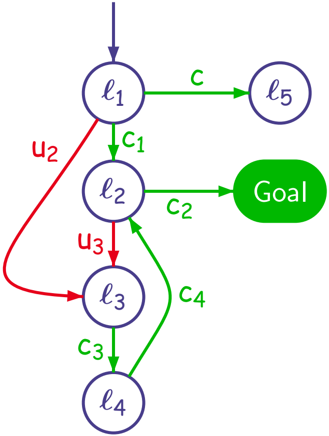

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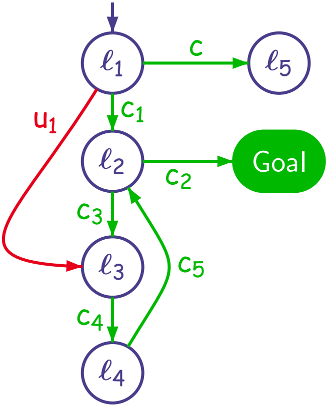
How to Solve Reachability Games?

Backward Computation of Winning States



$\overline{X}$ = complement of X

Controllable Predecessors:

$$\pi(X) = \left(cPred(X) \setminus uPred(\overline{X})\right)$$

Iterate π: $X_{i+1} = X_i \cup \pi(X_i)$

1. $X_0 = \{Goal\}$
2. $X_1 = \{Goal, \ell_2\}$
3. $X_2 = \{Goal, \ell_2, \ell_4\}$
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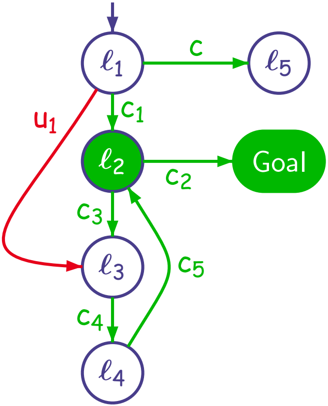

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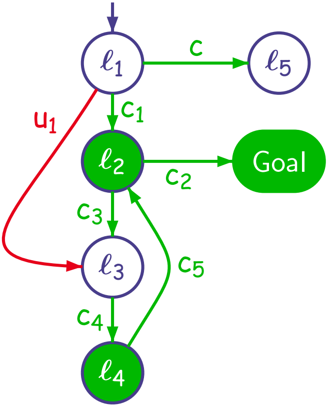

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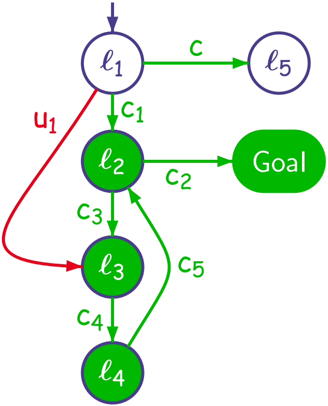

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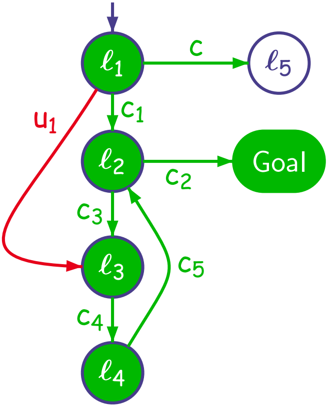

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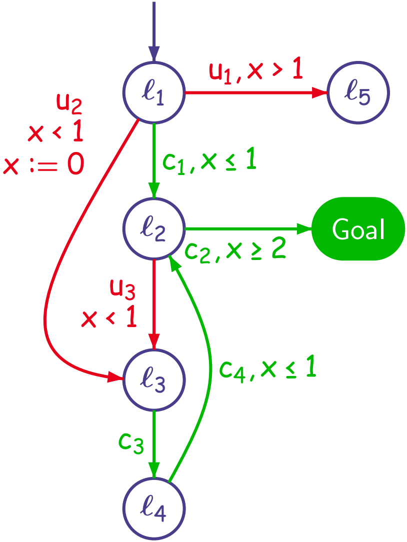
Reachability Control for Timed Games



Safe Time Elapsing:
When is it *safe* to let time elapse from q to q′ ?

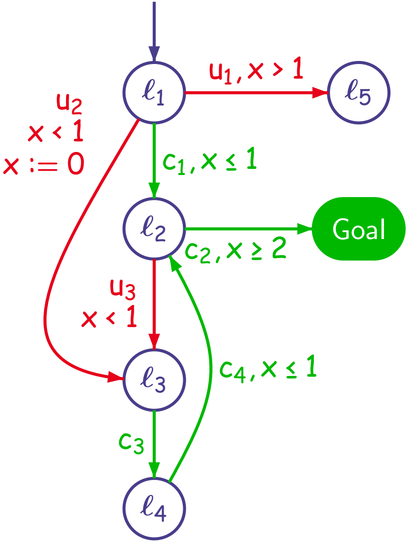
q $\qquad$ $q' \in X$

Controllable Predecessors:

$$\pi(X) = \mathrm{Pred}_t\left(X \cup \mathrm{cPred}(X), \mathrm{uPred}(\overline{X})\right)$$

▸ Timed Auto

Reachability Control for Timed Games



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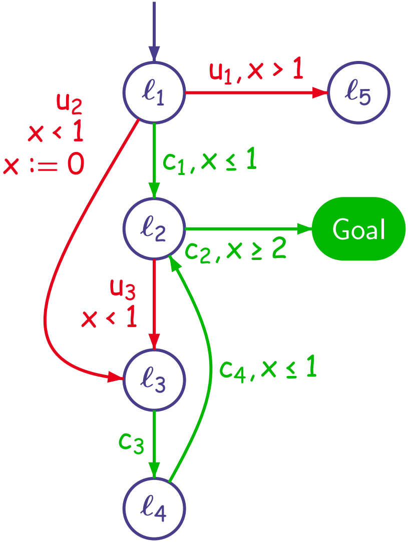
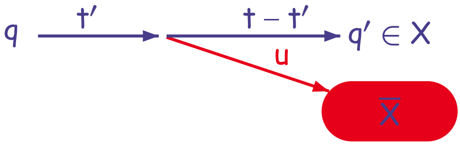
Timed Auto

Reachability Control for Timed Games



Safe Time Elapsing:
When is it *safe* to let time elapse from q to q' ?



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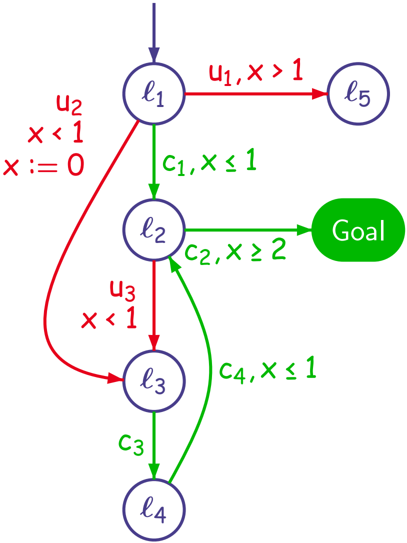

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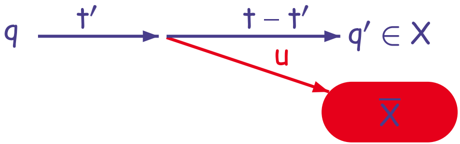

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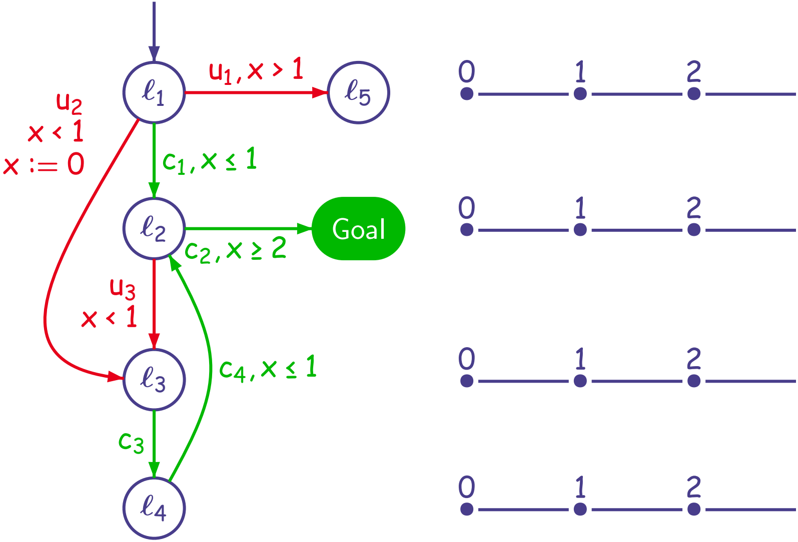

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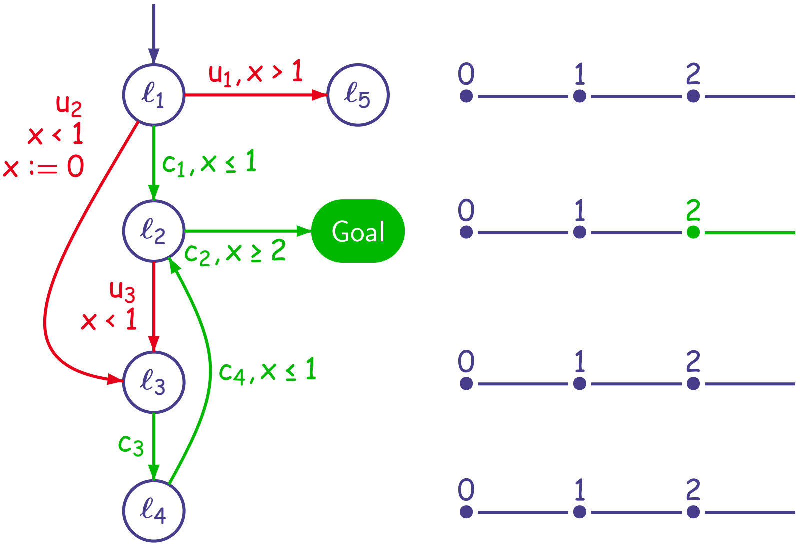

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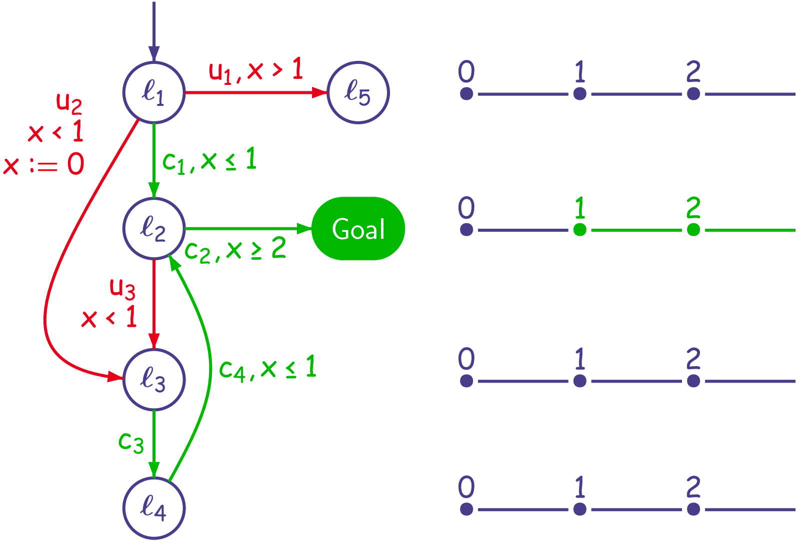

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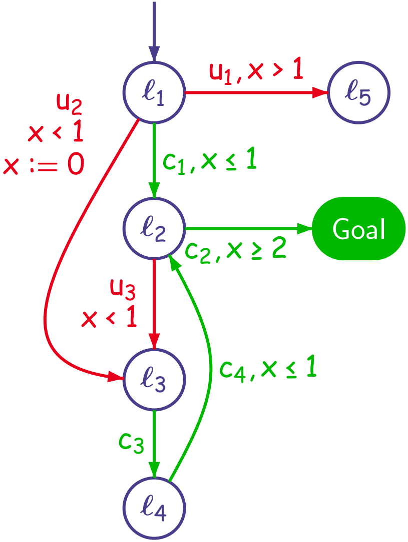

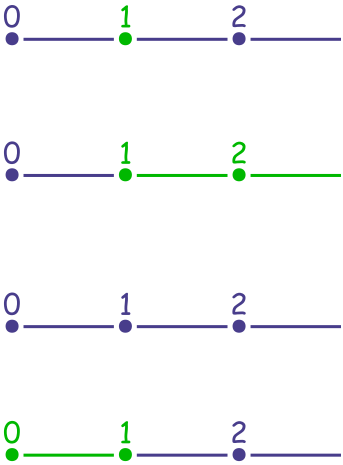

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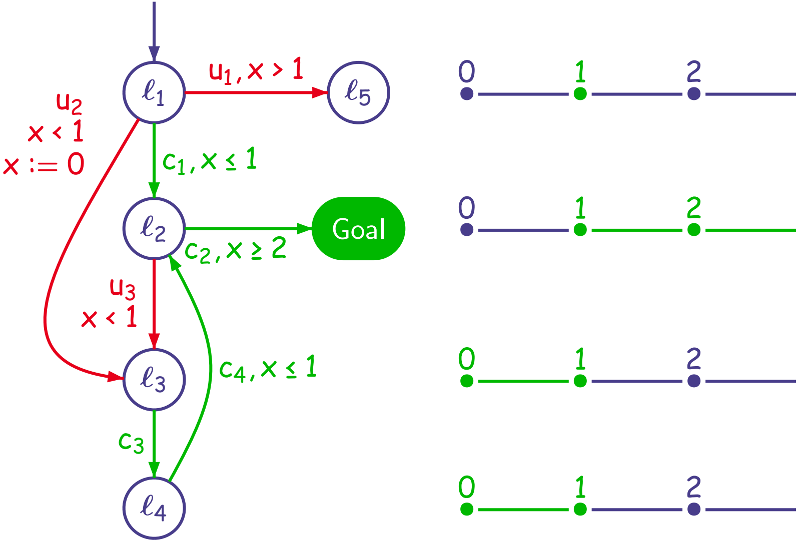

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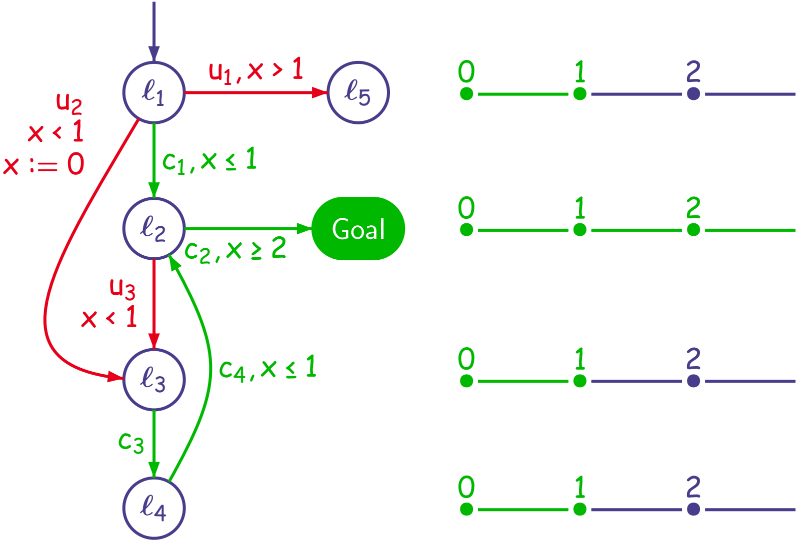

Summary of the Results for Reachability Control

Known Results for Timed (Game) Automata:

- Reachability in Timed Automata [Alur & Dill'94]
- Büchi Control for Timed Game Automata [Maler et al.'95]
- Time Optimal Control [Asarin & Maler'99]
- Optimal Control for Priced Timed Game Automata [Bouyer et al.'04a]
- Half on-the-fly algorithm [Altisen & Tripakis'99, Altisen & Tripakis'02]

New Results: True On-the-fly algorithm for reachability games

- Advantages: [Concur'05]
  - avoid constructing all backward & forward reachable states
  - allows for use of discrete variables (e.g. $i := i + 1$)
- Extends to Time-Optimal Control
- Extends to Partially Observable Games [ATVA'07]
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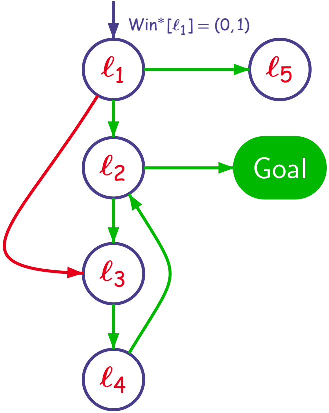
Liu & Smolka Algorithm [Liu & Smolka'98]



→ Current
→ Explored Forwards
Dashed = in Waiting List

Initialization:

```
    Passed ← {q0};
    Waiting ← {(q0, a, q') | a ∈ Act q --a--> q'};
    Win[q0] ← (q0 ∈ Goal ? 1 : 0);
    Depend[q0] ← ∅;
```

Main:

```
while ((Waiting ≠ ∅) ∧ Win[q0] ≠ 1)) do
    e = (q, a, q') ← pop(Waiting);
    if q' ∉ Passed then {
      Passed ← Passed ∪ {q'};
      Depend[q'] ← {(q, a, q')};
      Win[q'] ← (q' ∈ Goal ? 1 : 0);
      Waiting ← Waiting ∪ {(q', a, q'') | q' --a--> q''};
      Win*[q] ← (0, #{q --u-->});
      if Win[q'] then Waiting ← Waiting ∪ {e};
    }
    else (* reevaluate *)
      Win*[q] ← Update(Win*[q]) ;
      if (Win*[q] = (k, 0) ∧ k ≥ 1) then {
        Waiting ← Waiting ∪ Depend[q];
        Win[q] ← 1;
      }
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endwhile
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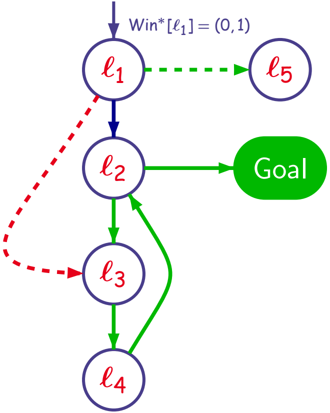
Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
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Dashed = in Waiting List

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Depend$[q_0] \leftarrow \emptyset$;

Main:
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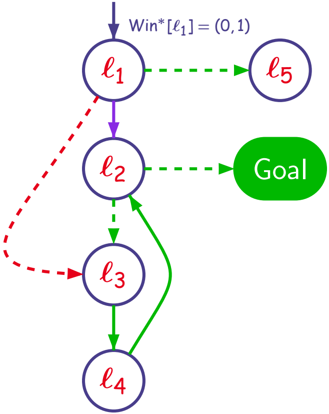

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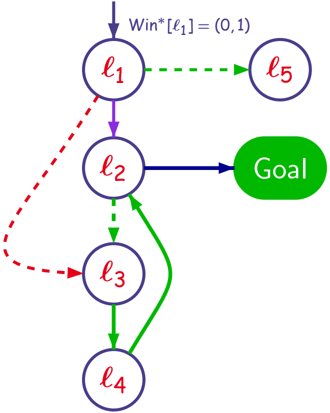

$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_2, c_2, Goal)$$

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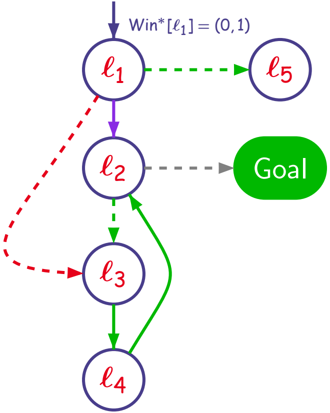

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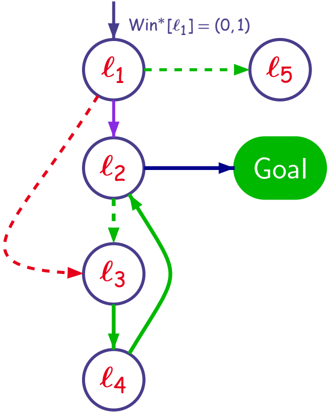

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Dashed = in Waiting List

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Initialization:

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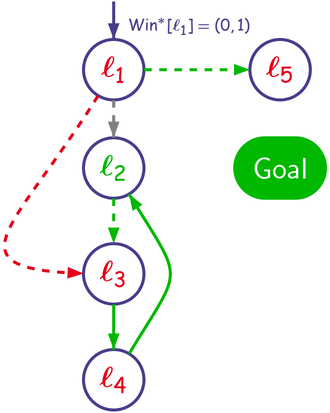

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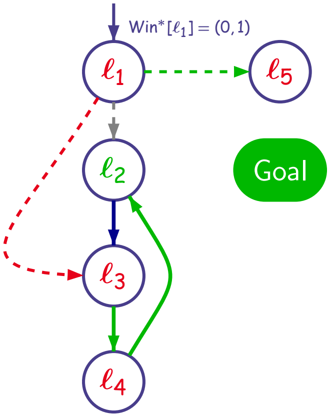

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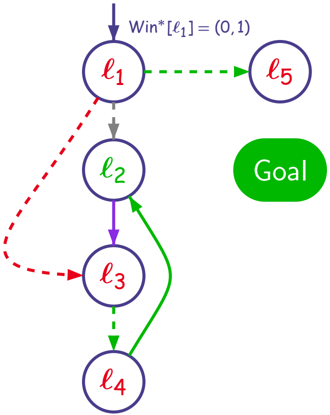

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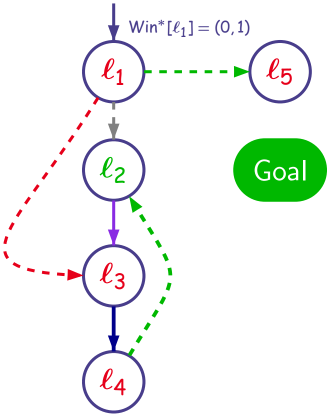

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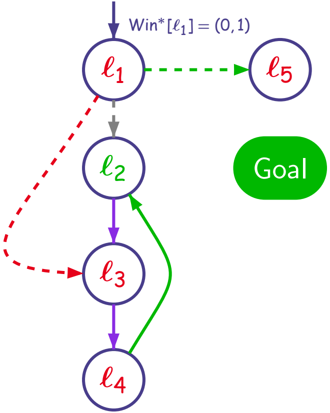

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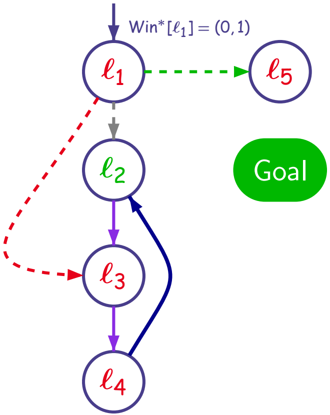
  if $q' \not\in$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in$ Goal ? 1 : 0);
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_4, c_5, \ell_2)$$

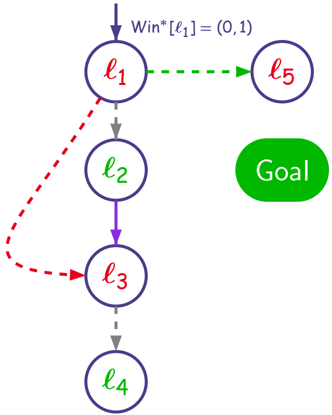
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in$ Goal ? 1 : 0);
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
 $e = (q, a, q') \leftarrow$ pop(Waiting);
 if $q' \notin$ Passed **then** {
  Passed $\leftarrow$ Passed $\cup \{q'\}$;
  Depend$[q'] \leftarrow \{(q, a, q')\}$;
  Win$[q'] \leftarrow (q' \in$ Goal ? 1 : 0);
  Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
  Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
  if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
 }
 else (* reevaluate *)
  Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
  if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
   Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
   Win$[q] \leftarrow 1$;
  }
  if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
 endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_4, c_5, \ell_2)$$

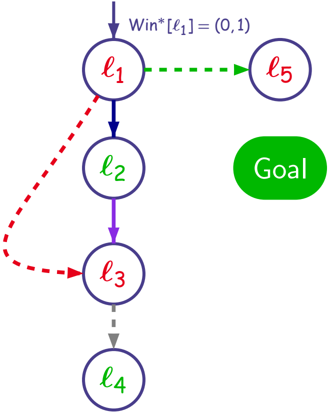
Initialization:

$\quad$ $Passed \leftarrow \{q_0\}$;
$\quad$ $Waiting \leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
$\quad$ $Win[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
$\quad$ $Depend[q_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1))$ **do**
$\quad$ $e = (q, a, q') \leftarrow pop(Waiting)$;
$\quad$ **if** $q' \not\in Passed$ **then** {
$\quad\quad$ $Passed \leftarrow Passed \cup \{q'\}$;
$\quad\quad$ $Depend[q'] \leftarrow \{(q, a, q')\}$;
$\quad\quad$ $Win[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
$\quad\quad$ $Waiting \leftarrow Waiting \cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
$\quad\quad$ $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
$\quad\quad$ **if** $Win[q']$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
$\quad$ }
$\quad$ **else (* reevaluate *)**
$\quad\quad$ $Win^*[q] \leftarrow$ **Update**$(Win^*[q])$;
$\quad\quad$ **if** $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
$\quad\quad\quad$ $Waiting \leftarrow Waiting \cup Depend[q]$;
$\quad\quad\quad$ $Win[q] \leftarrow 1$;
$\quad\quad$ }
$\quad\quad$ **if** $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
$\quad$ **endif**
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



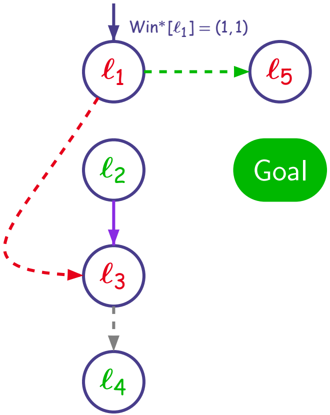
$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_1, c_1, \ell_2)$$

Initialization:
Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win[q_0] $\leftarrow (q_0 \in$ Goal ? 1 : 0);
Depend[q_0] $\leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ Win[q_0] $\neq$ 1)) **do**
  $e = (q, \alpha, q')$ $\leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend[q'] $\leftarrow \{(q, \alpha, q')\}$;
    Win[q'] $\leftarrow (q' \in$ Goal ? 1 : 0);
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win*[q] $\leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win[q'] **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win*[q] $\leftarrow$ **Update**(Win*[q]) ;
    if (Win*[q] $= (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend[q];
      Win[q] $\leftarrow$ 1;
    }
    if Win[q'] = 0 **then** Depend[q'] $\leftarrow$ Depend[q'] $\cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current

$\longrightarrow$ Explored Forwards

Dashed = in Waiting List

$$e = (\ell_1, c_1, \ell_2)$$

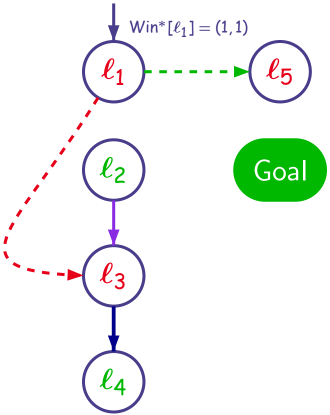
Initialization:

Passed $\longleftarrow \{q_0\}$;
Waiting $\longleftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \longleftarrow (q_0 \in$ Goal ? 1 : 0);
Depend$[q_0] \longleftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, a, q') \longleftarrow$ pop(Waiting);
  if $q' \not\in$ Passed **then** {
    Passed $\longleftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \longleftarrow \{(q, a, q')\}$;
    Win$[q'] \longleftarrow (q' \in$ Goal ? 1 : 0);
    Waiting $\longleftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
    Win$^*[q] \longleftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\longleftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \longleftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\longleftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \longleftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \longleftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

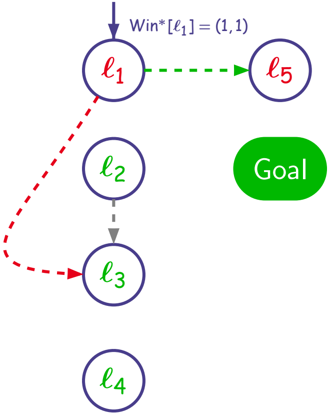
Initialization:

```
    Passed ← {q0};
    Waiting ← {(q0, a, q') | a ∈ Act q -a-> q'};
    Win[q0] ← (q0 ∈ Goal ? 1 : 0);
    Depend[q0] ← ∅;
```

Main:

```
while ((Waiting ≠ ∅) ∧ Win[q0] ≠ 1)) do
    e = (q, a, q') ← pop(Waiting);
    if q' ∉ Passed then {
      Passed ← Passed ∪ {q'};
      Depend[q'] ← {(q, a, q')};
      Win[q'] ← (q' ∈ Goal ? 1 : 0);
      Waiting ← Waiting ∪ {(q', a, q'') | q' -a-> q''};
      Win*[q] ← (0, #{q -u->});
      if Win[q'] then Waiting ← Waiting ∪ {e};
    }
    else (* reevaluate *)
      Win*[q] ← Update(Win*[q]) ;
      if (Win*[q] = (k, 0) ∧ k ≥ 1) then {
        Waiting ← Waiting ∪ Depend[q];
        Win[q] ← 1;
      }
      if Win[q'] = 0 then Depend[q'] ← Depend[q'] ∪ {e};
    endif
endwhile
```

Liu & Smolka Algorithm [Liu & Smolka'98]



→ Current
→ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

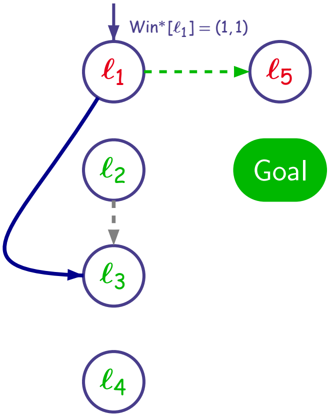
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in$ Goal ? 1 : 0);
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, a, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, a, q')\}$;
    Win$[q'] \leftarrow (q' \in$ Goal ? 1 : 0);
    Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
    Win*$[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win*$[q] \leftarrow$ **Update**(Win*$[q]$) ;
    if (Win*$[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

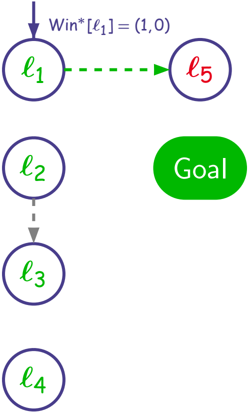
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in$ Goal ? $1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while $((\text{Waiting} \neq \emptyset) \wedge \text{Win}[q_0] \neq 1))$ **do**
 $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
 if $q' \not\in$ Passed **then {**
  Passed $\leftarrow$ Passed $\cup \{q'\}$;
  Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
  Win$[q'] \leftarrow (q' \in$ Goal ? $1 : 0)$;
  Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
  $\text{Win}^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
  if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
 }
 else (* reevaluate *)
  $\text{Win}^*[q] \leftarrow$ **Update**$(\text{Win}^*[q])$;
  if $(\text{Win}^*[q] = (k, 0) \wedge k \geq 1)$ **then {**
   Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
   Win$[q] \leftarrow 1$;
  }
  if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
 endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



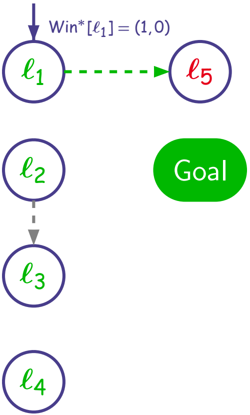
$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

Initialization:
Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in$ Goal ? 1 : 0);
Depend$[q_0] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
$e = (q, \alpha, q') \leftarrow$ pop(Waiting);
if $q' \notin$ Passed **then** {
Passed $\leftarrow$ Passed $\cup \{q'\}$;
Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
Win$[q'] \leftarrow (q' \in$ Goal ? 1 : 0);
Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
}
else (* reevaluate *)
Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
Win$[q] \leftarrow 1$;
}
if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



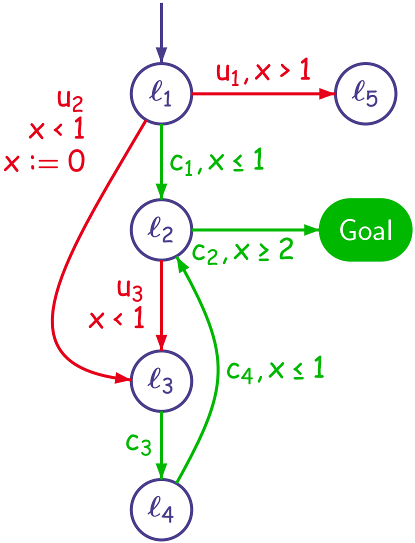
$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

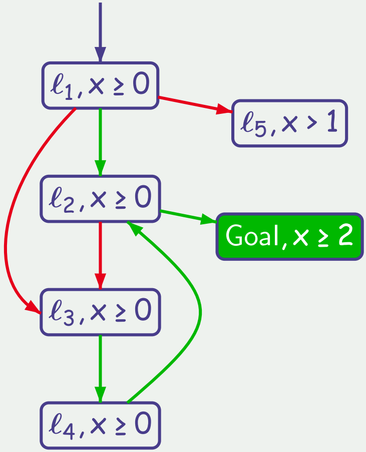
Initialization:
 Passed $\leftarrow \{q_0\}$;
 Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
 Win$[q_0] \leftarrow (q_0 \in$ Goal ? 1 : 0);
 Depend$[q_0] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
 $e = (q, a, q') \leftarrow$ pop(Waiting);
 if $q' \notin$ Passed **then** {
  Passed $\leftarrow$ Passed $\cup \{q'\}$;
  Depend$[q'] \leftarrow \{(q, a, q')\}$;
  Win$[q'] \leftarrow (q' \in$ Goal ? 1 : 0);
  Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
  Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
  if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
 }
 else (* reevaluate *)
  Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
  if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
   Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
   Win$[q] \leftarrow 1$;
  }
  if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
 endif
endwhile

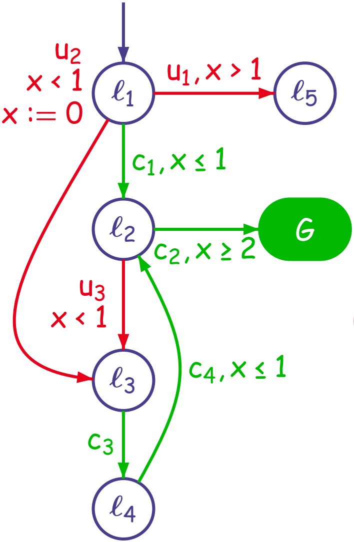
On-The-Fly Algorithm for Timed Games (1)



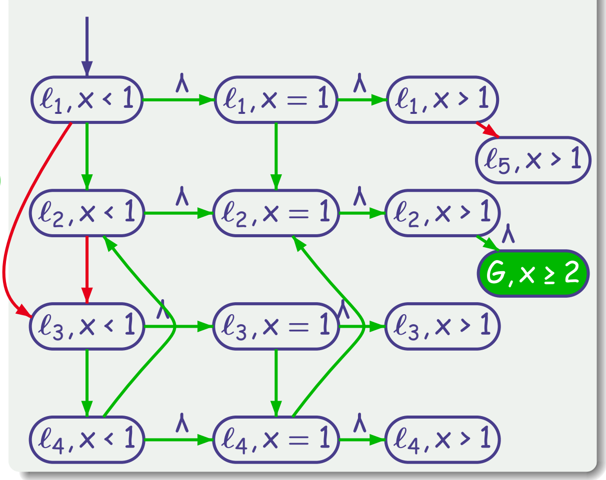
Using the Simulation Graph



Second Try (2) [Altisen & Tripakis'99, Altisen & Tripakis'02]



Stable Partitionning



Towards a True On-The-Fly Algorithm

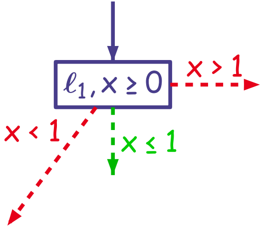
To Do:

- Write a Symbolic version of Liu & Smolka
- Use Symbolic states and Transitions
- Apply this to Timed Games

Key issues to be adressed:

- Symbolic States can be partially winning
  compared to finite state games where 0 or 1
- When to propagate backwards ?
- Termination, Complexity ?

Liu & Smolka for Timed Games



Initialization:

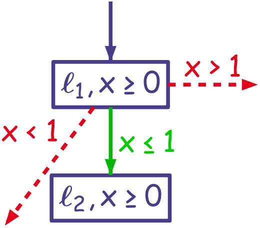
Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{\text{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ($(\ell_0, 0) \notin$ Win$[S_0]$)) **do**
 $e = (S, a, S') \leftarrow$ pop(Waiting);
 if $S' \notin$ Passed **then**
  Passed $\leftarrow$ Passed $\cup \{S'\}$;
  Depend$[S'] \leftarrow \{(S, a, S')\}$;
  Win$[S'] \leftarrow S' \cap (\{\text{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
  Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
  if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
 else **(* reevaluate *)**
  Win$^* \leftarrow$ Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
   $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
  if (Win$[S] \subsetneq$ Win*) **then**
   Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
  Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
 endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

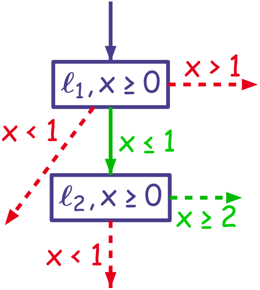
  Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
  Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
  Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
  Depend$[S_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge ((\ell_0, 0) \notin Win[S_0]))$ **do**
  $e = (S, a, S') \leftarrow pop(Waiting)$;
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend$[S'] \leftarrow \{(S, a, S')\}$;
    Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    $Win^* \leftarrow Pred_t(Win[S] \cup \bigcup_{S \xrightarrow{c} T} cPred(Win[T]),$
      $\bigcup_{S \xrightarrow{u} T} uPred(T \setminus Win[T])) \cap S$;
    if $(Win[S] \subsetneq Win^*)$ **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow Win^*$;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

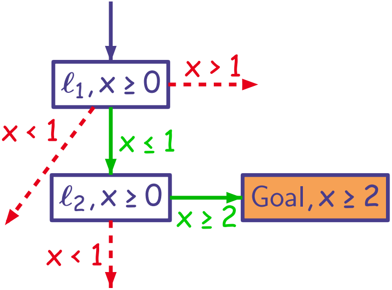
Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ (($\ell_0, 0) \notin$ Win$[S_0]$)) **do**
  $e = (S, a, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend$[S'] \leftarrow \{(S, a, S')\}$;
    Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    $Win^* \leftarrow Pred_t(Win[S] \cup \bigcup_{S \xrightarrow{c} T} cPred(Win[T]), \bigcup_{S \xrightarrow{u} T} uPred(T \setminus Win[T])) \cap S$;
    if (Win$[S] \subsetneq Win^*$) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow Win^*$;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, \alpha, S') \mid S' = Post_\alpha(S_0)^{\nearrow}\}$;
Win[S_0] $\leftarrow S_0 \cap (\{\text{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
Depend[S_0] $\leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ((ℓ_0, 0) $\notin$ Win[S_0])) **do**
  $e = (S, \alpha, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend[S'] $\leftarrow \{(S, \alpha, S')\}$;
    Win[S'] $\leftarrow S' \cap (\{\text{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', \alpha, S'') \mid S'' = Post_\alpha(S')^{\nearrow}\}$;
    if Win[S'] $\neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    Win$^* \leftarrow$Pred$_t$(Win[S]$\cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win[T]),
      $\bigcup_{S \xrightarrow{u} T}$ uPred(T \ Win[T])) $\cap$ S;
    if (Win[S] $\subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend[S]; Win[S] $\leftarrow$ Win*;
    Depend[S'] $\leftarrow$ Depend[S'] $\cup \{e\}$;
  endif
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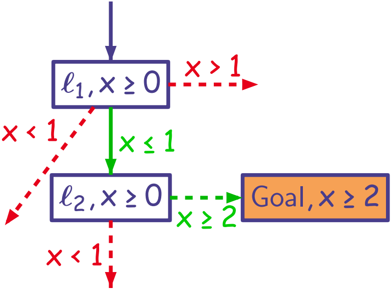

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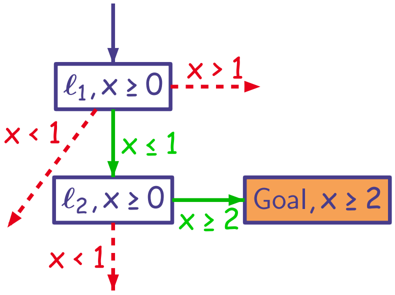

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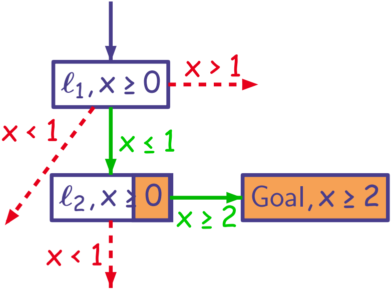

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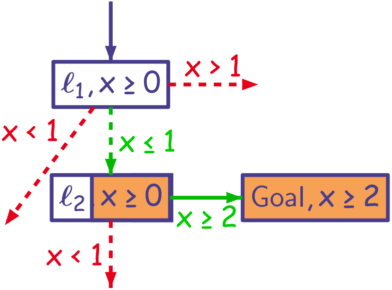

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$\quad\quad$ **if** Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
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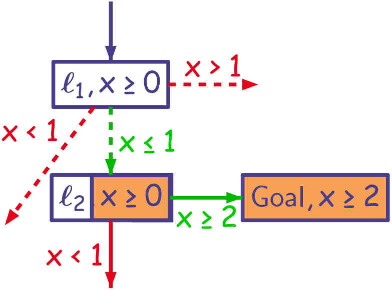

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Main:

while $((Waiting \neq \emptyset) \wedge ((\ell_0, 0) \not\sqsubseteq Win[S_0]))$ **do**
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  if $S' \not\in$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    $Depend[S'] \leftarrow \{(S, \alpha, S')\}$;
    $Win[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', \alpha, S'') \mid S'' = Post_\alpha(S')^{\nearrow}\}$;
    if $Win[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
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    $Win^* \leftarrow Pred_t(Win[S] \cup \bigcup_{S \xrightarrow{c} T} cPred(Win[T]), \bigcup_{S \xrightarrow{u} T} uPred(T \setminus Win[T])) \cap S$;
    if $(Win[S] \subsetneq Win^*)$ **then**
      Waiting $\leftarrow$ Waiting $\cup$ $Depend[S]$; $Win[S] \leftarrow Win^*$;
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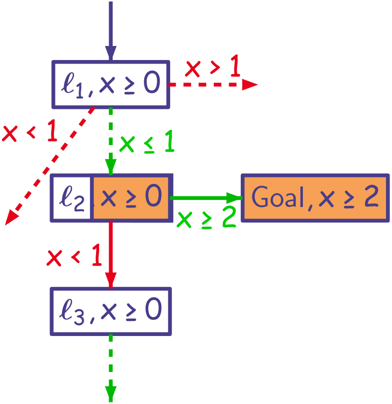

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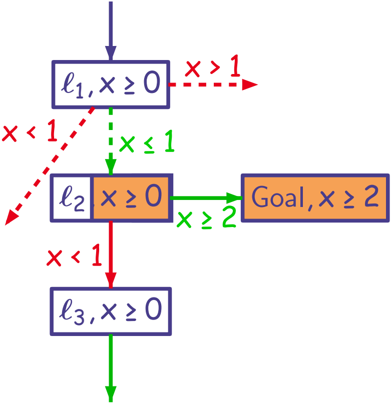

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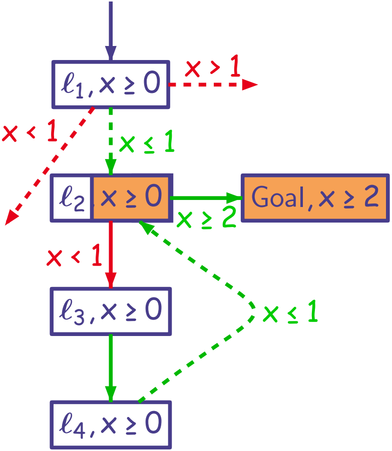

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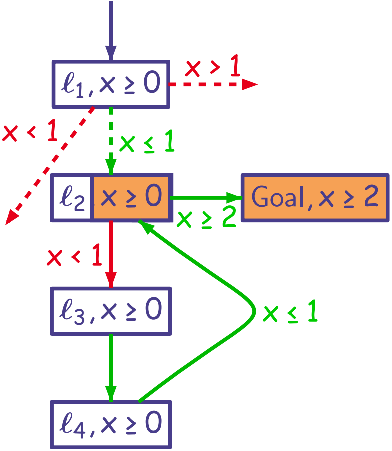

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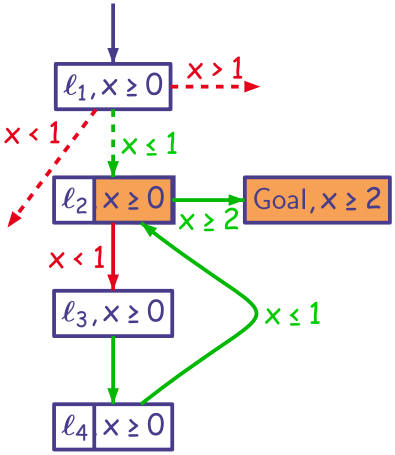

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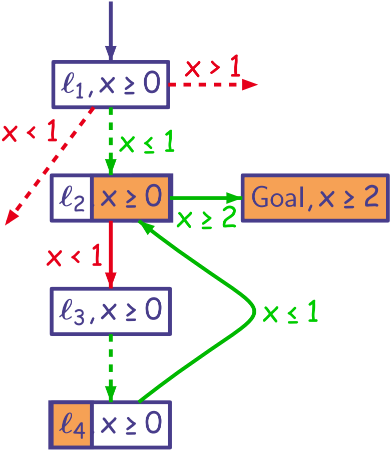

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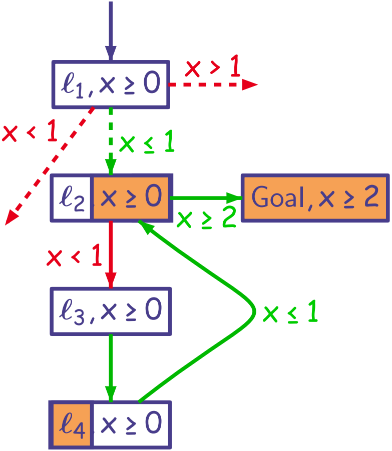

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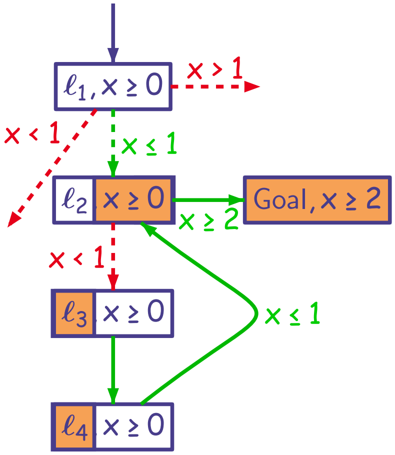
Skip algorithm

Liu & Smolka for Timed Games



Skip algorithm

Initialization:

Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, \alpha, S') \mid S' = Post_\alpha(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ((ℓ_0, 0) $\notin$ Win$[S_0]$)) **do**
  $e = (S, \alpha, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend$[S'] \leftarrow \{(S, \alpha, S')\}$;
    Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', \alpha, S'') \mid S'' = Post_\alpha(S')^{\nearrow}\}$;
    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* **reevaluate** *)
    Win* $\leftarrow$Pred$_t$(Win$[S]\cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

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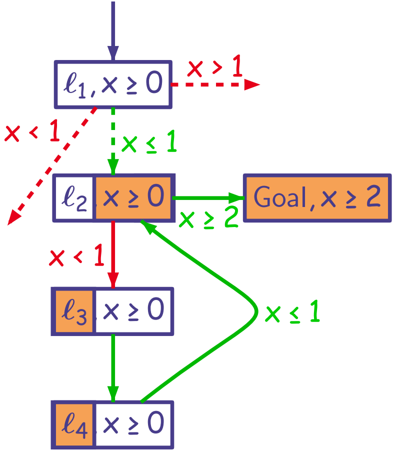

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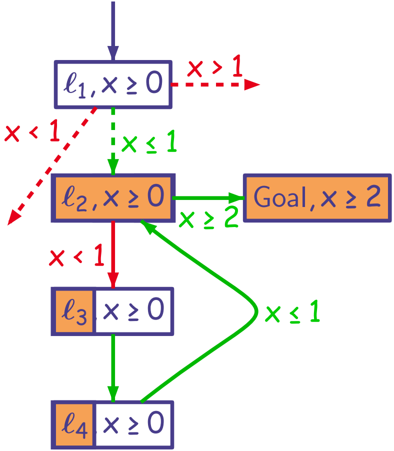

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Main:

while $((\mathrm{Waiting} \neq \emptyset) \wedge ((\ell_0, 0) \notin \mathrm{Win}[S_0]))$ **do**
  $e = (S, \alpha, S') \leftarrow \mathrm{pop}(\mathrm{Waiting})$;
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
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    Waiting $\leftarrow$ Waiting $\cup \{(S', \alpha, S'') \mid S'' = Post_\alpha(S')^{\nearrow}\}$;
    if $\mathrm{Win}[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    $\mathrm{Win}^* \leftarrow \mathrm{Pred}_t(\mathrm{Win}[S] \cup \bigcup_{S \xrightarrow{c} T} \mathrm{cPred}(\mathrm{Win}[T]), \bigcup_{S \xrightarrow{u} T} \mathrm{uPred}(T \setminus \mathrm{Win}[T])) \cap S$;
    if $(\mathrm{Win}[S] \subsetneq \mathrm{Win}^*)$ **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend[S]; $\mathrm{Win}[S] \leftarrow \mathrm{Win}^*$;
    $\mathrm{Depend}[S'] \leftarrow \mathrm{Depend}[S'] \cup \{e\}$;
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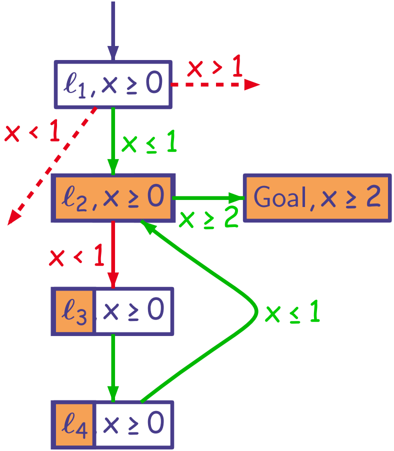

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Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

```
while ((Waiting ≠ ∅) ∧ ((ℓ0, 0) ∉ Win[S0])) do
    e = (S, α, S') ← pop(Waiting);
    if S' ∉ Passed then
        Passed ← Passed ∪ {S'};
        Depend[S'] ← {(S, α, S')};
        Win[S'] ← S' ∩ ({Goal} × R^X_{≥0});
        Waiting ← Waiting ∪ {(S', α, S'') | S'' = Post_α(S')↗};
        if Win[S'] ≠ ∅ then Waiting ← Waiting ∪ {e};
    else (* reevaluate *)
        Win* ←Pred_t(Win[S] ∪ ⋃_{S -c-> T} cPred(Win[T]),
                             ⋃_{S -u-> T} uPred(T \ Win[T])) ∩ S;
        if (Win[S] ⊊ Win*) then
            Waiting ← Waiting ∪ Depend[S]; Win[S] ← Win*;
        Depend[S'] ← Depend[S'] ∪ {e};
    endif
endwhile
```

Skip algorithm

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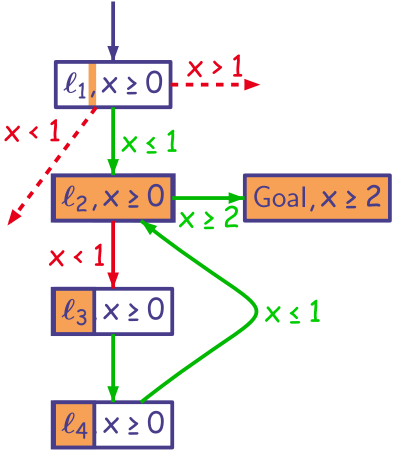

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    if ($\mathrm{Win}[S] \subsetneq \mathrm{Win}^*$) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend[S]; $\mathrm{Win}[S] \leftarrow \mathrm{Win}^*$;
    $\mathrm{Depend}[S'] \leftarrow \mathrm{Depend}[S'] \cup \{e\}$;
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Skip algorithm

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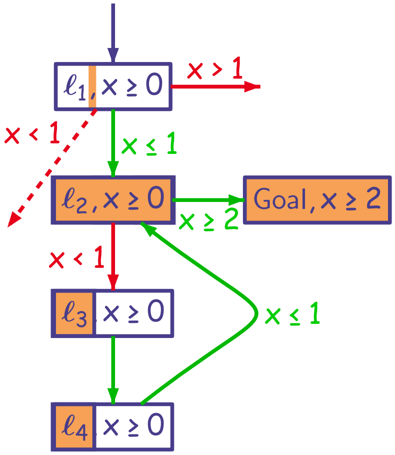

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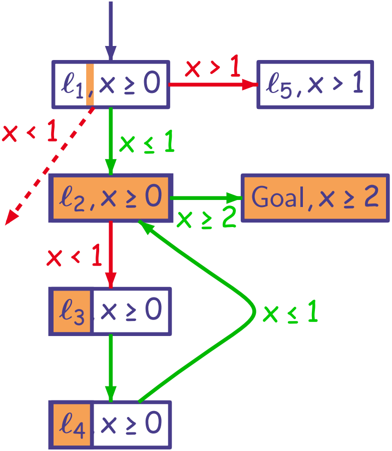

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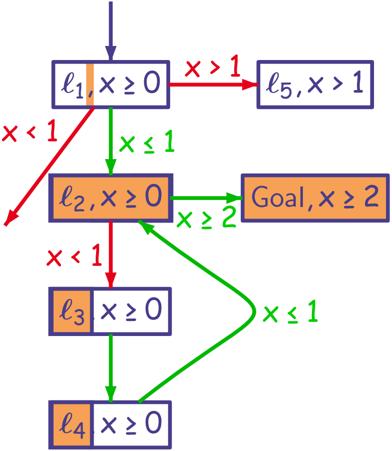

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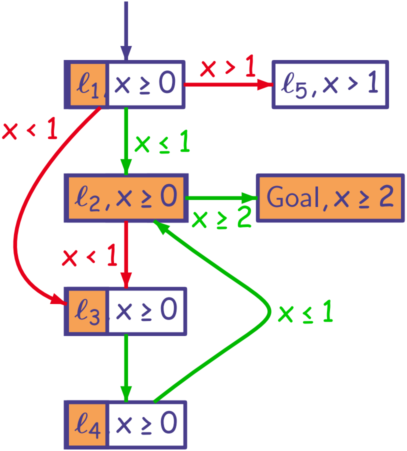

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$\quad$ **endif**
endwhile

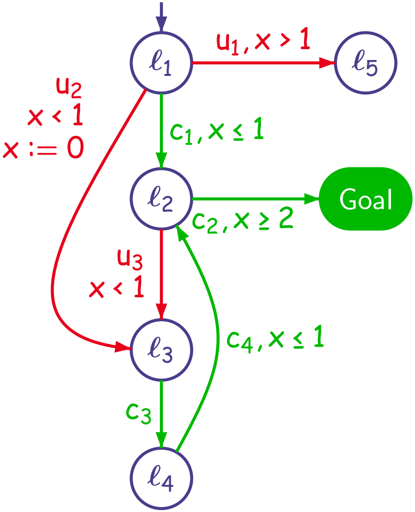
Skip algorithm

Summary of the Results [Concur'05]

- A True on-the-fly algorithm for reachability control
- Winning Strategies can be computed
- Termination
  A symbolic edge (S, a, T) will be at most (1+ # regions(T)) times in *Waiting* list
- Complexity
  A region may be in many symbolic states
  Our algorithm: Not linear in the size of the region graph
  hence not theoretically optimal
- However ... seems good in practice with **UPPAAL-TiGA**

Download at `http://www.cs.aau.dk/~adavid/tiga/`

Time Optimality for Free



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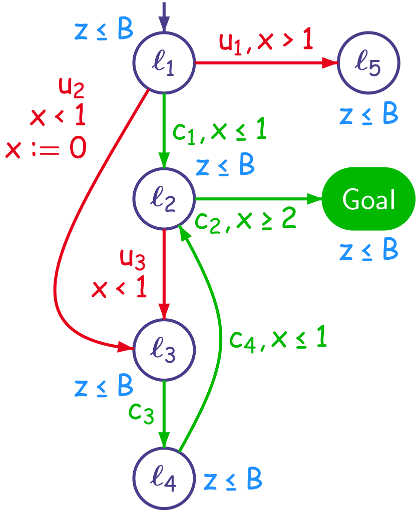
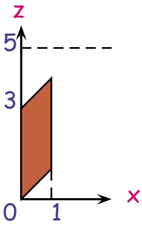


Assume:

- The initial state is winning
- We know an upper bound B of the (optimal) time needed to reach Goal

To compute the optimal time:

- Add a clock z (unconstrained at the beginning)
- Add a global invariant $z \leq B$



Next:

- Control of Timed Systems: Basics
- Control of Discrete Event Systems
- Control of Timed Systems
- Advanced Subjects
- **Conclusion**

Conclusion

- Recent Research Results:
  - Implementability of controllers
  - Optimality of controllers
  - Efficient algorithms for solving Timed Games
  - Control under Partial-Observation
- Ongoing work:
  - Efficient Algorithms for Safety, Büchi games
  - Data Structures for optimal control
  - Optimal control for infinite schedules
  - Synthesis of robust controllers
  - Abstraction/Refinement for synthesis of controllers

Merci !

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Timed Automata [Alur & Dill'94]

A Timed Automaton $\mathcal{A}$ is a tuple $(L, \ell_0, Act, X, \mathrm{inv}, \longrightarrow)$ where:

- L is a finite set of locations
- ℓ_0 is the initial location
- X is a finite set of clocks
- Act is a finite set of actions
- $\longrightarrow$ is a set of transitions of the form $\ell \xrightarrow{g,a,R} \ell'$ with:
  - $\ell, \ell' \in L$,
  - $a \in Act$
  - a guard g which is a clock constraint over X
  - a reset set R which is the set of clocks to be reset to 0

Clock constraints are boolean combinations of $x \sim k$ with $x \in C$ and $k \in \mathbb{Z}$ and $\sim \in \{\leq, <\}$.

Semantics of Timed Automata

Let $\mathcal{A} = (L, \ell_0, Act, X, inv, \longrightarrow)$ be a Timed Automaton.

A state (ℓ, v) of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$

The semantics of $\mathcal{A}$ is a Timed Transition System $S_{\mathcal{A}} = (Q, q_0, Act \cup \mathbb{R}_{\geq 0}, \longrightarrow)$ with:

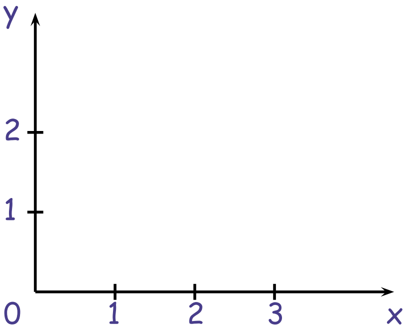
- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$
- $\longrightarrow$ consists in:

discrete transition: $(\ell, v) \xrightarrow{a} (\ell', v') \iff \begin{cases} \exists\, \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A} \\ v \models g \\ v' = v[r \leftarrow 0] \\ v' \models inv(\ell') \end{cases}$

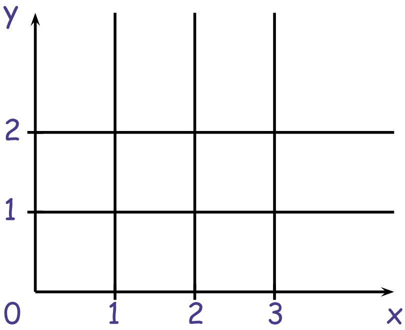
delay transition: $(\ell, v) \xrightarrow{d} (\ell, v + d) \iff d \in \mathbb{R}_{\geq 0} \wedge v + d \models inv(\ell)$

The Region Abstraction

[Alur & Dill'94]



The Region Abstraction

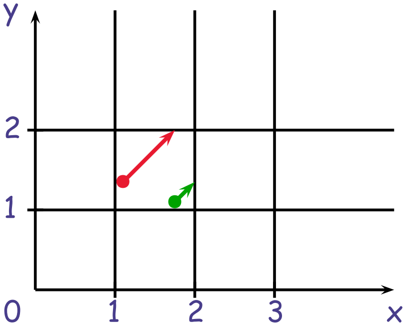


Build an equivalence relation which is of finite index and is:

- "compatible" with clock constraints ($g ::= x \sim c \quad g \wedge g$)

  $r, r' \in R \implies \forall$ constraints $g, \quad r \models g \iff r' \models g$

The Region Abstraction



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- "compatible" with time elapsing

  $r, r' \in R \implies$ same delay successor regions

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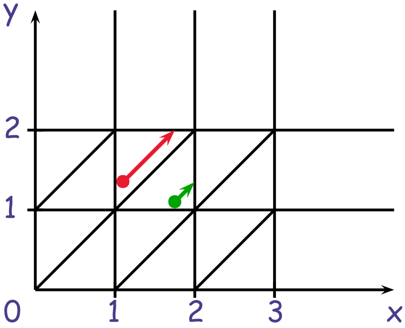

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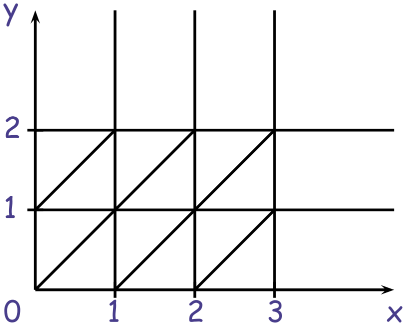

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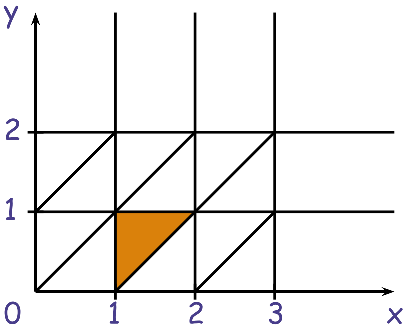

region defined by

$I_x =]1;2[$ $I_y =]0;1[$

$\{x\} < \{y\}$

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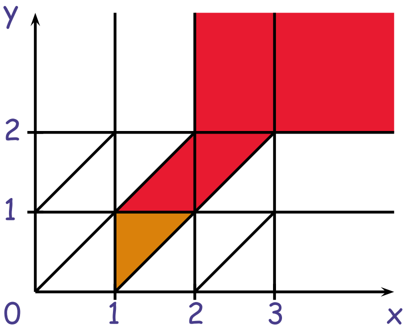
The Region Abstraction [Alur & Dill'94]



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delay successors

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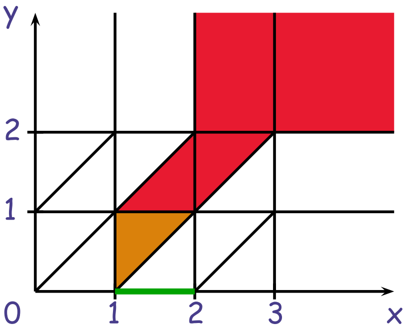
[Alur & Dill'94]



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delay successors

successor by reset

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  $r, r' \in R \implies$ same delay successor regions

The Region Automaton

- For each transition $\ell \xrightarrow{g,a,C:=0} \ell'$ of the TA
- Build transitions in the region automaton RA: $(\ell, R) \xrightarrow{a} (\ell', R')$ if:
  - there exists R'' a delay successor of R s.t.
  - R'' satisfies the guard g ($R'' \subseteq [\![g]\!]$)
  - $R''[C \leftarrow 0]$ is included in R'

a TA and its region automaton RA are time-abstract bisimilar

- The region automaton is finite
- Language accepted by the RA = untimed language accepted by the TA

  a timed word $w = (a, 1.2)(b, 3.4)(a, 6.256)$; untimed$(w) = aba$
- Language Emptyness can be decided on the RA

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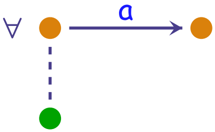
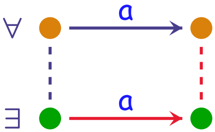
Time-abstract bisimulation



$$\begin{array}{ccccccc}
(\ell_0, v_0) & \xrightarrow{a_1, t_1} & (\ell_1, v_1) & \xrightarrow{a_2, t_2} & (\ell_2, v_2) & \xrightarrow{a_3, t_3} & \cdots \\
\updownarrow & & \updownarrow & & \updownarrow & & \\
(\ell_0, R_0) & \xrightarrow{a_1} & (\ell_1, R_1) & \xrightarrow{a_2} & (\ell_2, R_2) & \xrightarrow{a_3} & \cdots
\end{array}$$

with $v_i \in R_i$ for all i.

Time-abstract bisimulation



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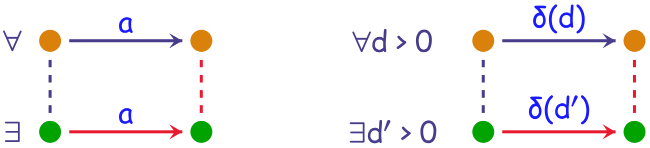

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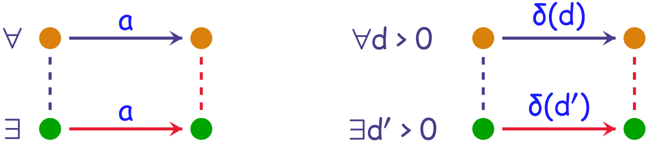

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$$\updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow$$

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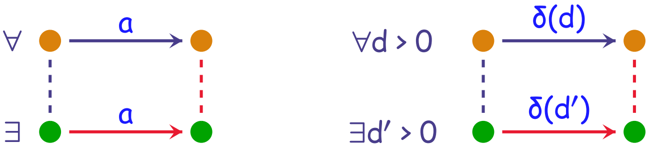

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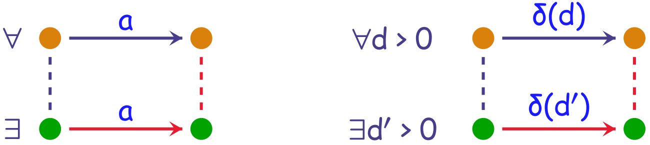

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Definition (Outcome in Timed Games)

Let $G = (L, \ell_0, Act, X, E, inv)$ be a TGA and f a strategy over G. The **outcome** $\mathrm{Outcome}((\ell, v), f)$ of f from configuration (ℓ, v) in G is the subset of $\mathrm{Runs}((\ell, v), G)$ defined inductively by:

- $(\ell, v) \in \mathrm{Outcome}((\ell, v), f)$,
- if $\rho \in \mathrm{Outcome}((\ell, v), f)$ then $\rho' = \rho \xrightarrow{e} (\ell', v') \in \mathrm{Outcome}((\ell, v), f)$ if $\rho' \in \mathrm{Runs}((\ell, v), G)$ and one of the following three conditions hold:
  1. $e \in Act_u$,
  2. $e \in Act_c$ and $e = f(\rho)$,
  3. $e \in \mathbb{R}_{\geq 0}$ and $\forall 0 \leq e' < e, \exists (\ell'', v'') \in (L \times \mathbb{R}_{\geq 0}^X)$ s.t. $last(\rho) \xrightarrow{e'} (\ell'', v'') \wedge f(\rho \xrightarrow{e'} (\ell'', v'')) = \lambda$.
- an infinite run ρ is in $\in \mathrm{Outcome}((\ell, v), f)$ if all the finite prefixes of ρ are in $\mathrm{Outcome}((\ell, v), f)$.

States & Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ is the set of states of the TGA
  $q = (\ell, v) \in Q$
- Discrete predecessors of $X \subseteq Q$ by an action a:
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X is a state predicate

- $cPred(X) = \bigcup_{c \in Act_c} Pred^c(X)$ $uPred(X) = \bigcup_{u \in Act_u} Pred^u(X)$
  cPred and uPred are effectively computable
- $Pred_\delta(X, Y)$: Time controllable predecessors of X avoiding Y:

q $q' \in X$

$Pred_\delta(X, Y)$ is effectively computable for state predicates X, Y

- Controllable Predecessors Operator for Timed Games

$$\pi_\delta(X) = Pred_\delta\left(cPred(X), uPred(\overline{X})\right)$$

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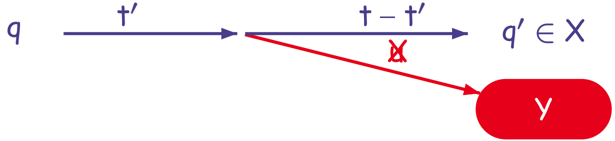

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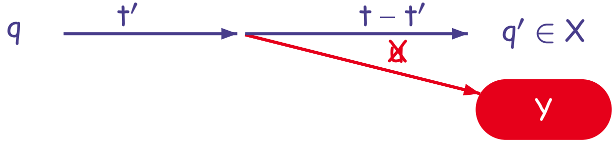

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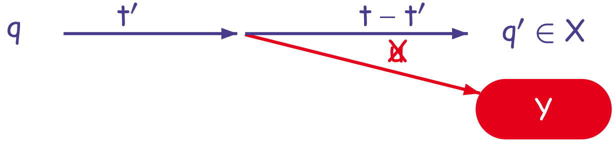

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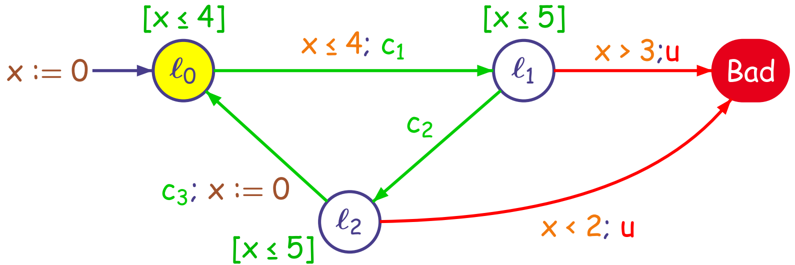

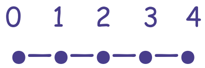

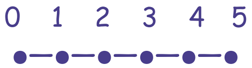

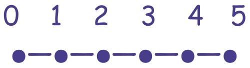

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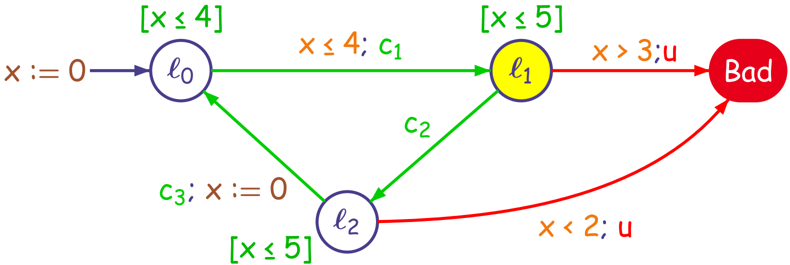

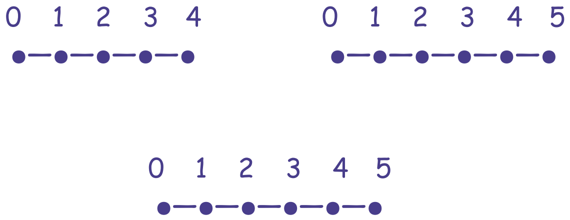

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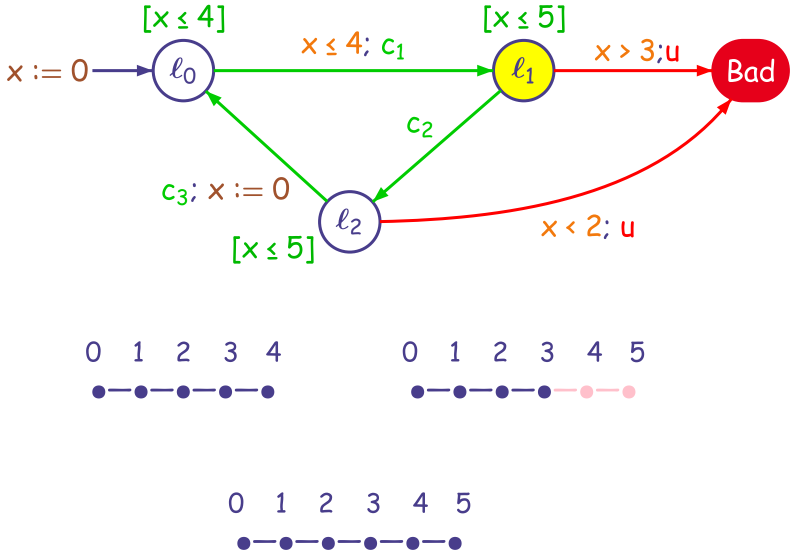

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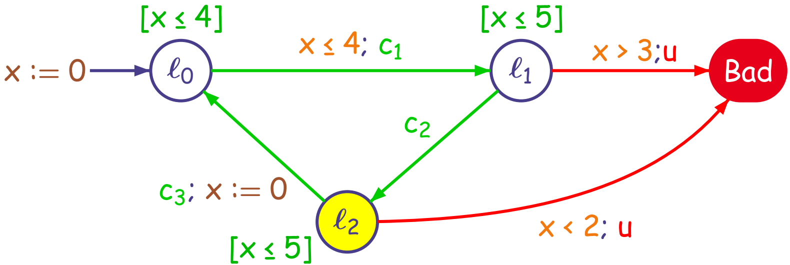

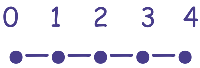

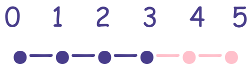

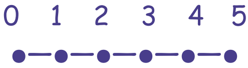

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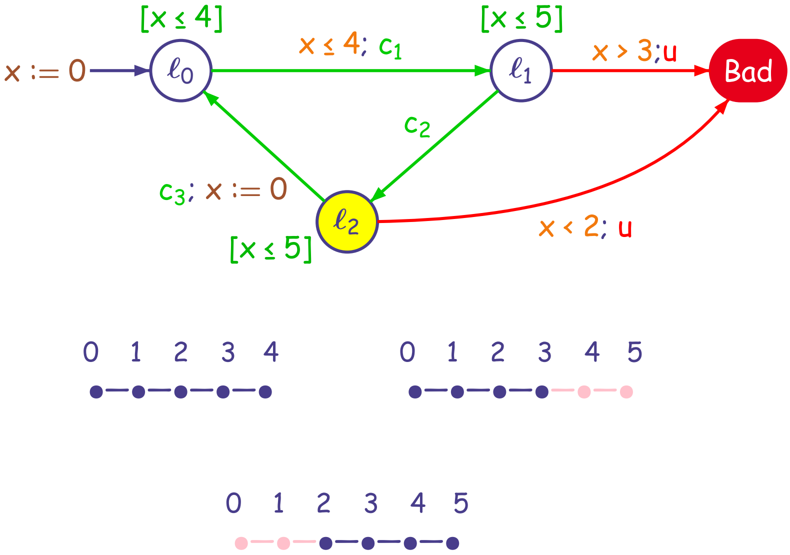

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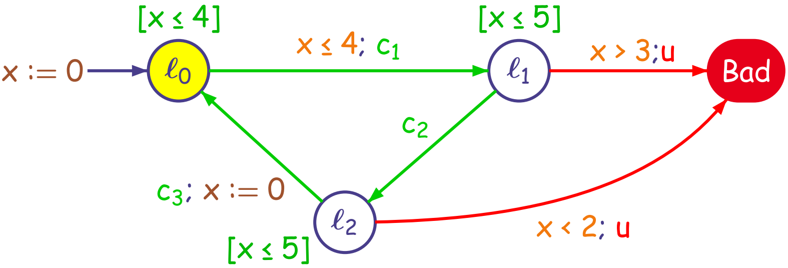

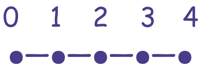

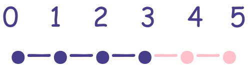

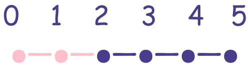

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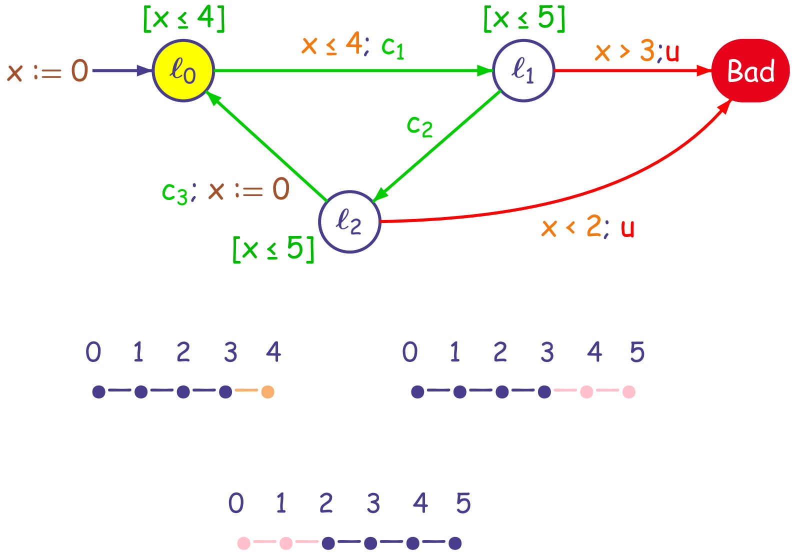

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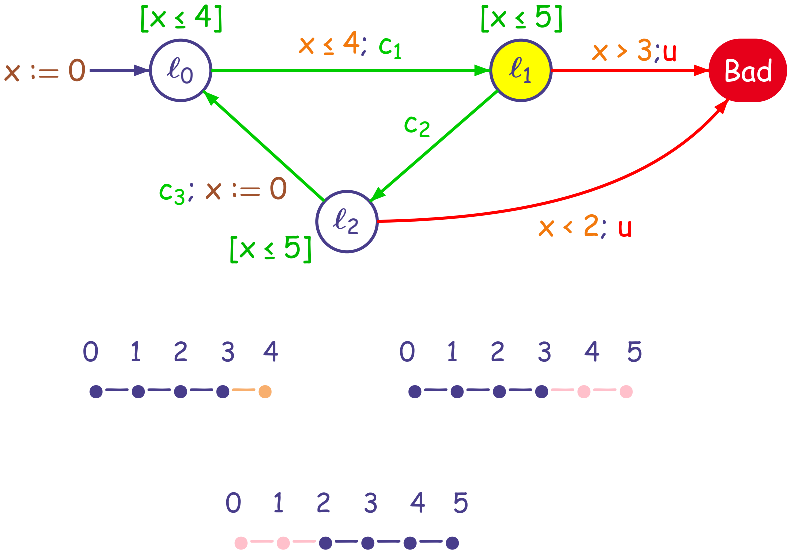

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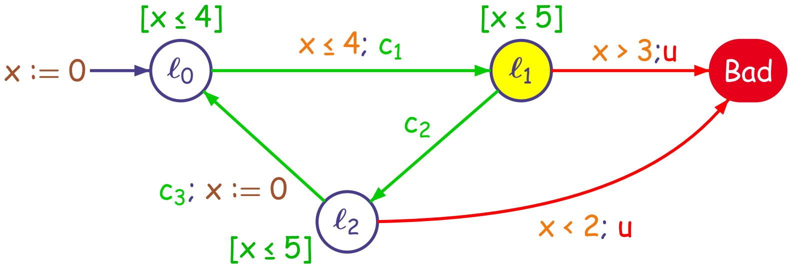

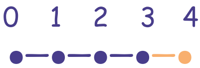

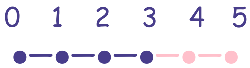

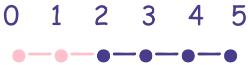

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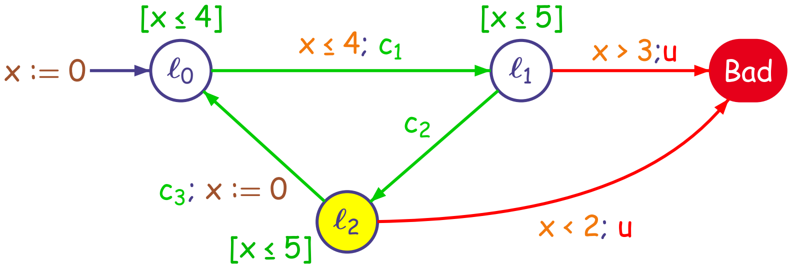

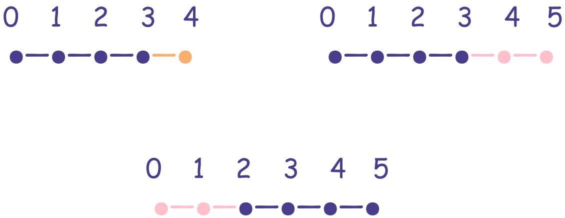

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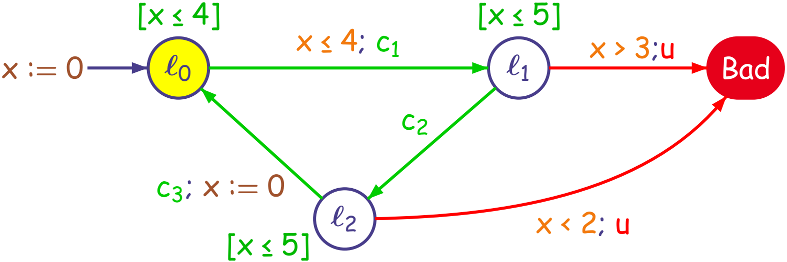

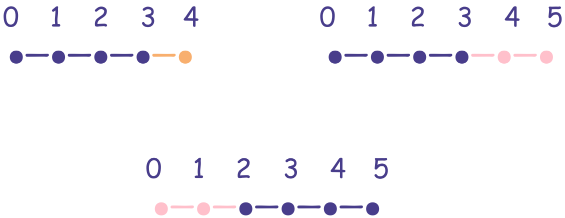

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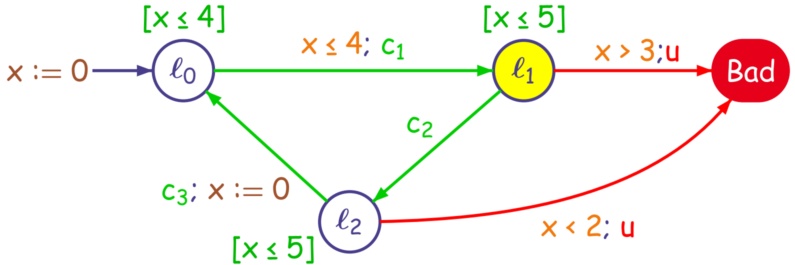

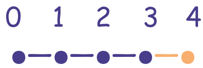

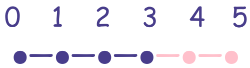

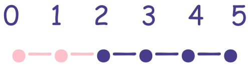

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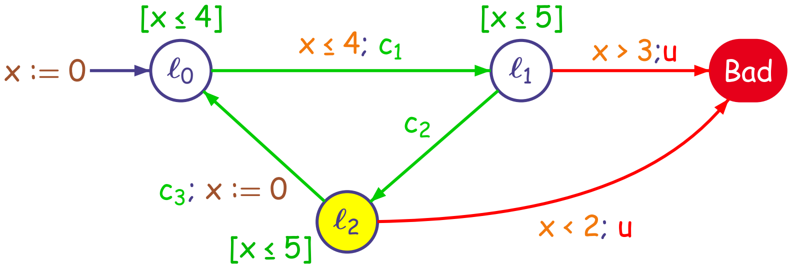

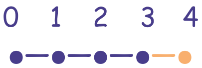

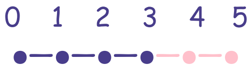

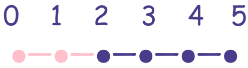

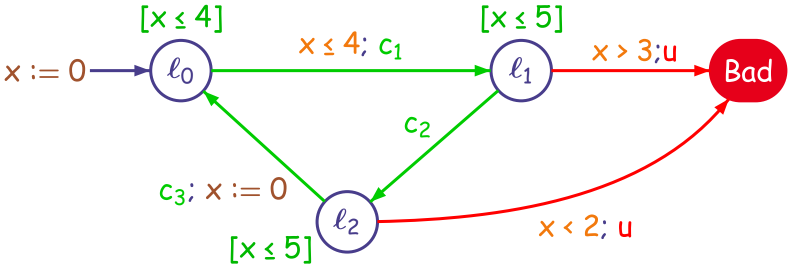

Winning States

$(\ell_0, 0 \leq x \leq 3)$
$(\ell_1, 0 \leq x \leq 3)$
$(\ell_2, 2 \leq x \leq 5)$

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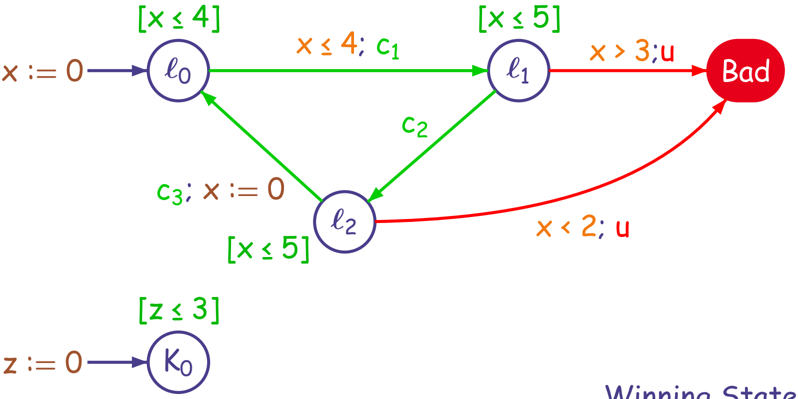

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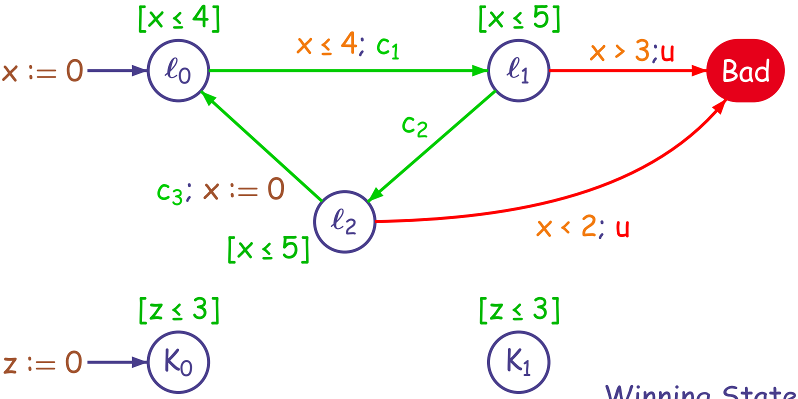

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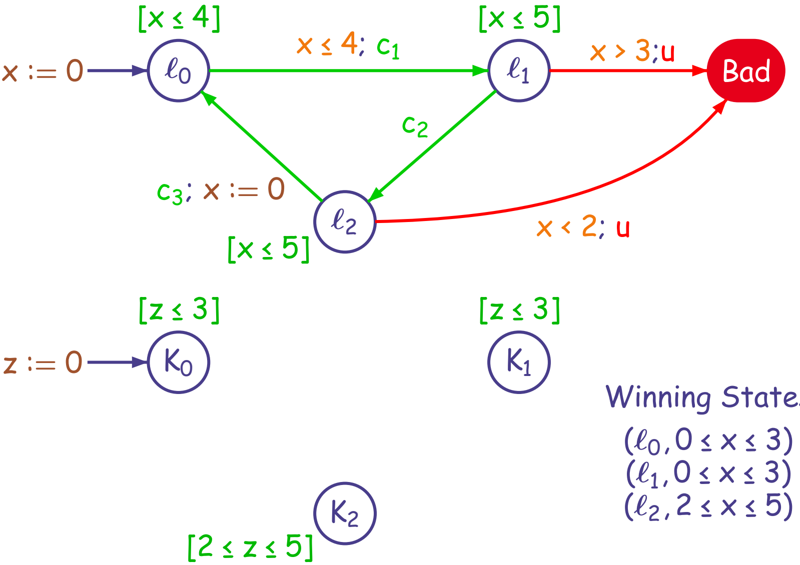

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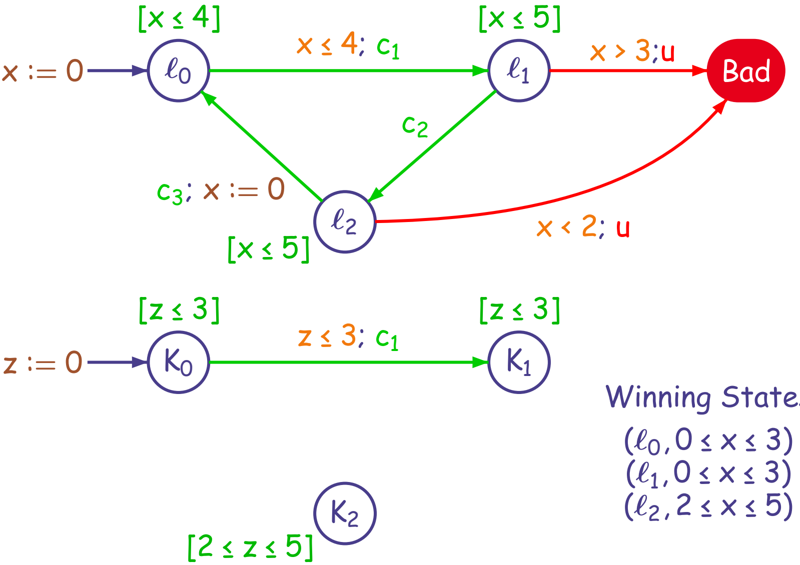

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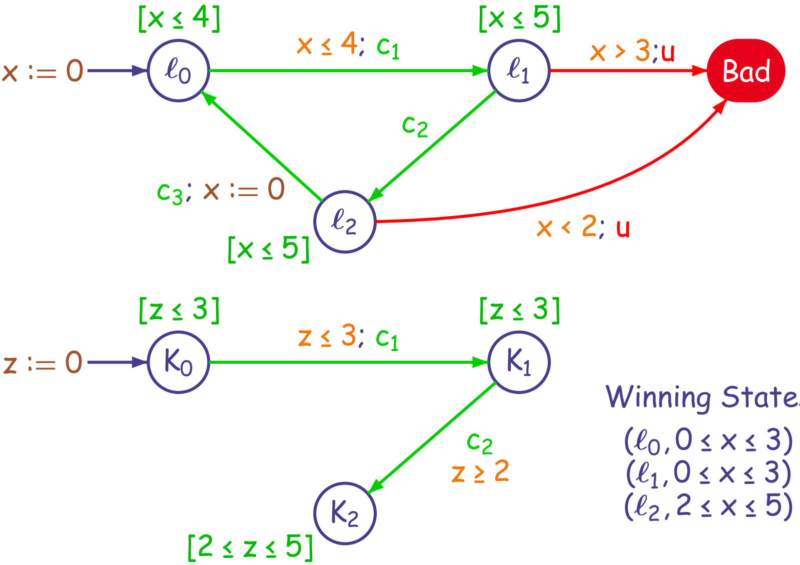

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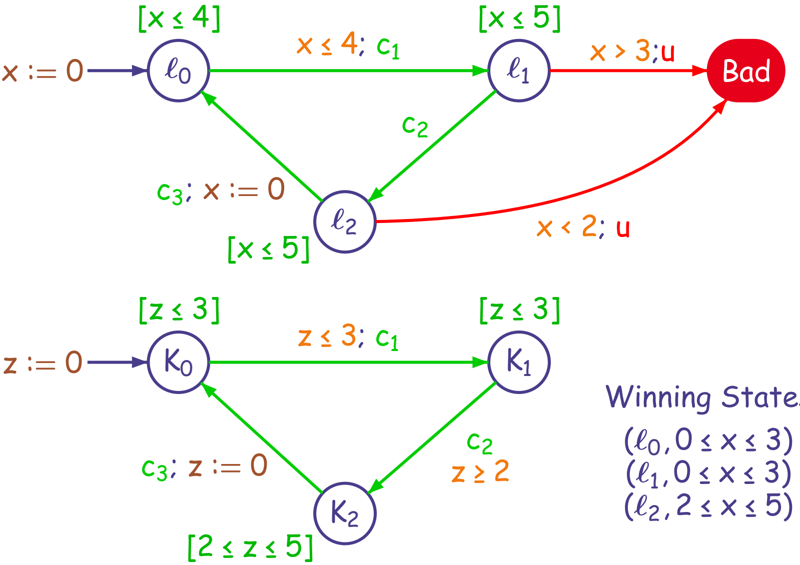

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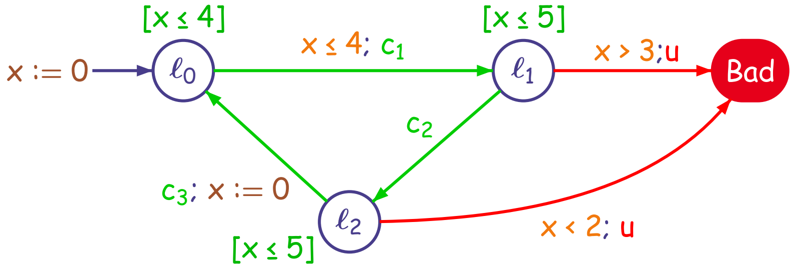

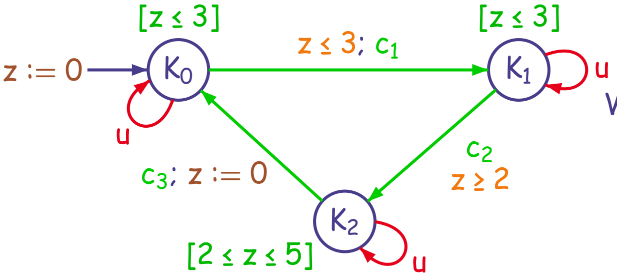

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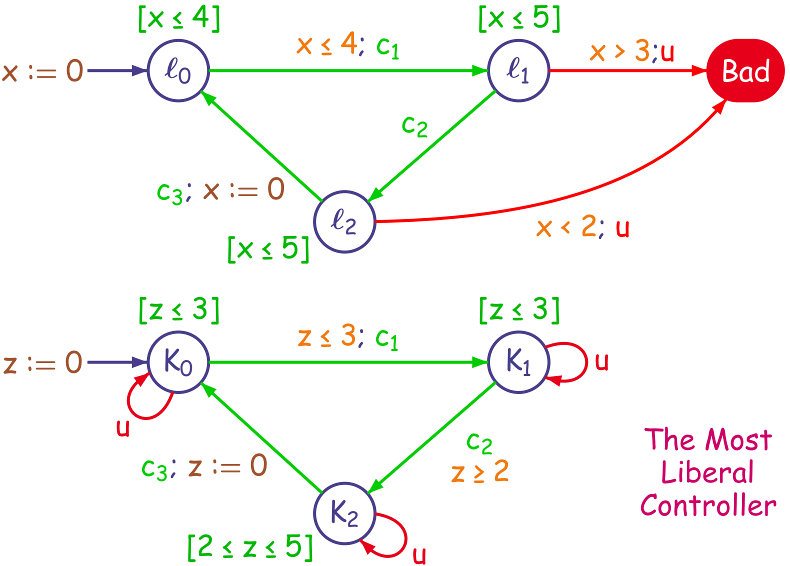

The Most Liberal Controller

Existence of Cost Independent Strategies

Let A be a RPTGA such that:

- guards of u actions are strict
- guards on c actions are large

There is an optimal cost independent strategy

Is it necessary ?

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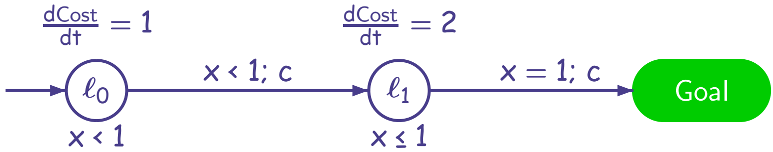
No Optimal Strategy

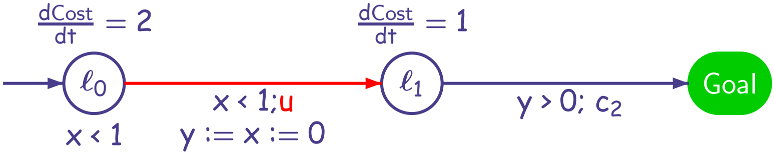


- define f_ε with $0 < \varepsilon < 1$ by:
  in ℓ_0: $f(\ell_0, x < 1 - \varepsilon) = \lambda$, $f(\ell_0, 1 - \varepsilon \leq x < 1) = c$
  in ℓ_1: $f(\ell_1, x < 1) = \lambda$, $f(\ell_1, x = 1) = c$
  $Cost(f_\varepsilon) = (1 - \varepsilon) + 2.\varepsilon = 1 + \varepsilon$ and $OptCost = 1$.
- given $\varepsilon > 0$, there is a sub-optimal strategy f_ε such that

$$|Cost((\ell_0, \vec{0}), f_\varepsilon) - OptCost((\ell_0, \vec{0}), G)| < \varepsilon$$

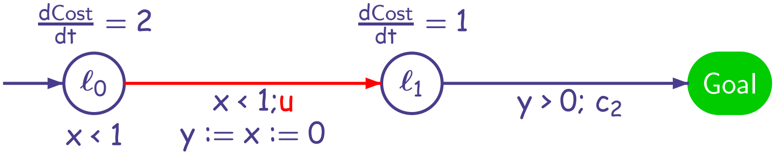
- New problem: given ε, compute such an f_ε strategy.

No Optimal Cost-Independent Strategy



- Optimal cost is 2

No Optimal Cost-Independent Strategy

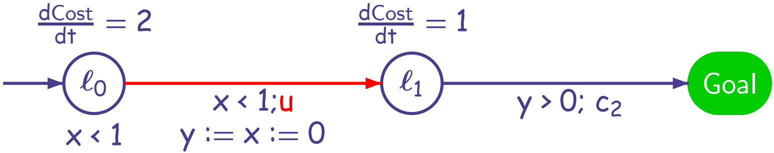


- Optimal cost is 2
- An optimal winning cost-dependent strategy f:
  $f(\ell_1, -, cost < 2) = \lambda$ and $f(\ell_1, -, cost = 2) = c_2$
  assume u taken at time $(1 - \delta_0)$:

$$Cost(f, (\ell_0, 0)) = 2 \cdot (1 - \delta_0) + \delta_1 = 2$$

because according to f we have $\delta_1 = 2 \cdot \delta_0$

No Optimal Cost-Independent Strategy



- Optimal cost is 2
- assume $\exists$ f^* cost-independent: f^* must wait in ℓ_1 at least ε
  assume u taken at time $(1 - \delta)$:

$$\mathrm{Cost}(f^*, (\ell_0, 0)) = 2 \cdot (1 - \delta) + \varepsilon$$

Take $\delta = \frac{\varepsilon}{4}$: $\mathrm{Cost}(f^*, (\ell_0, 0)) = 2 + \frac{\varepsilon}{2}$ and $\mathrm{OptCost}(f^*) = 2 + \varepsilon$

Related Work for Optimal Control

- [La Torre et al.'02]
  - Acyclic Priced Timed Game Automata
  - Recursive definition of optimal cost
  - Computation of the infimum of the optimal cost
    i.e. OptCost = 2 could mean that it is 2 or $2 + \varepsilon$
  - No strategy synthesis
- [Alur et al.'04] (ICALP'04)
  - Bounded optimality: optimal cost within k steps
  - Complexity bound: exponential in k and #states of the PTGA
  - No bound for the more general optimal problem
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  - No strategy synthesis
- Our work [Bouyer et al.'04a]:
  - Run-based definition of optimal cost
  - We can decide whether $\exists$ an optimal strategy
  - We can effectively synthesize an optimal strategy (if one exists)
  - We can prove structural properties of optimal strategies
  - Applies to Linear Hybrid Game (Automata)

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