Temporal logics for multi-agent systems

Nicolas Markey
LSV – ENS Cachan

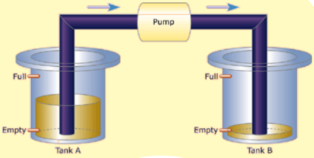
(based on joint works with Thomas Brihaye,
Arnaud Da Costa-Lopes, François Laroussinie)

« Formalisation des Activités Concurrentes »

Toulouse, 16 April 2014

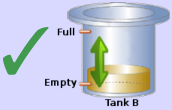
Model checking and synthesis

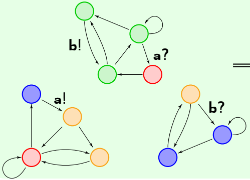
system:



[http://www.embedded.com]

propriété



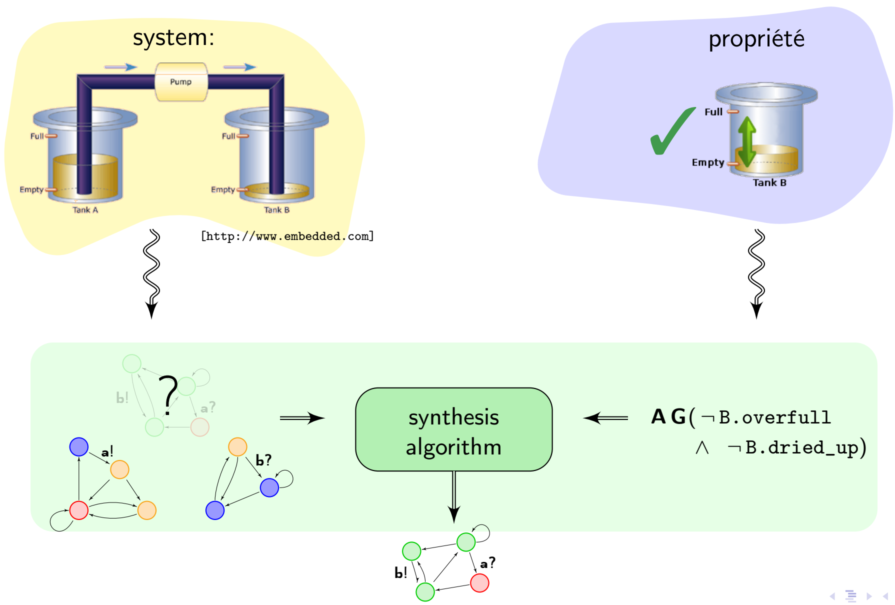


model-checking algorithm

$\mathbf{A}\,\mathbf{G}(\neg \mathtt{B.overfull} \wedge \neg \mathtt{B.dried\_up})$

yes/no

Model checking and synthesis



Outline of the presentation

1. Introduction
2. Basics of CTL and ATL
   - expressing properties of reactive systems
   - efficient verification algorithms
3. Temporal logics for multi-agent systems
   - specifying properties of complex interacting systems
   - expressive power of ATL_{sc}
   - translation into Quantified CTL (QCTL)
   - algorithms for ATL_{sc}
4. Conclusions and future works

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Computation-Tree Logic (CTL)

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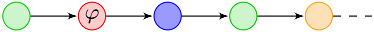
X φ 

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φ **U** ψ 

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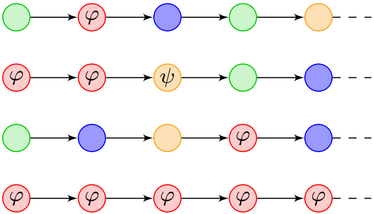

$\mathbf{X}\,\varphi$	"next φ"
$\varphi\ \mathbf{U}\ \psi$	"φ until ψ"
$\mathtt{true}\ \mathbf{U}\ \varphi \equiv \mathbf{F}\,\varphi$	"eventually φ"
$\neg\ \mathbf{F}\ \neg\varphi \equiv \mathbf{G}\,\varphi$	"always φ"

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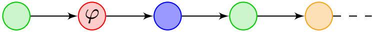
$\mathbf{X}\,\varphi$ 

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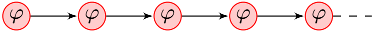
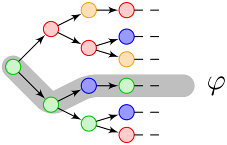
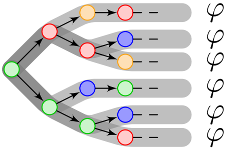
$\neg\,\mathbf{F}\,\neg\varphi \equiv \mathbf{G}\,\varphi$ 

"always φ"

- **path quantifiers:**

$\mathbf{E}\varphi$ 

$\mathbf{A}\varphi$ 

Examples of CTL and CTL* formulas

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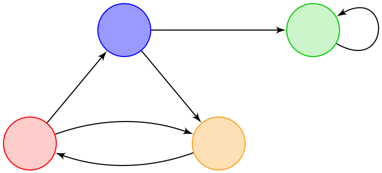

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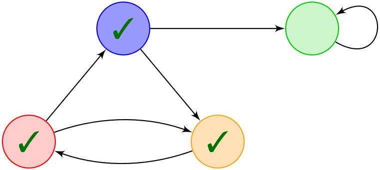

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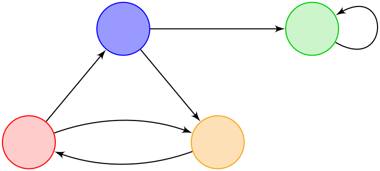
$\mathbf{E}\,\mathbf{G}(\neg \bigcirc \wedge \mathbf{E}\,\mathbf{F} \bigcirc)$ there is a path along which $\bigcirc$ is always reachable, but never reached



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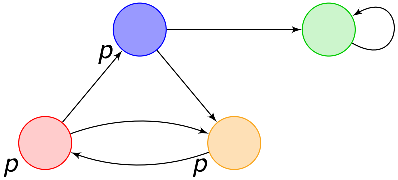

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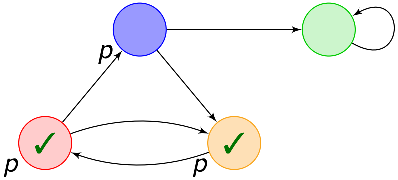

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Theorem ([CE81,QS82])

CTL model checking is PTIME*-complete.*

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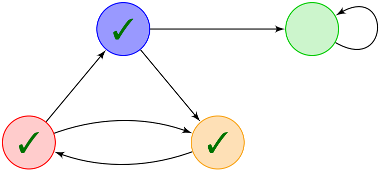
E G F  there is a path visiting  infinitely many times



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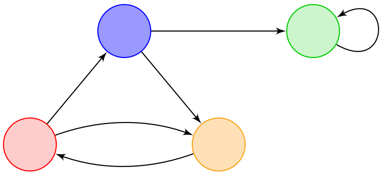

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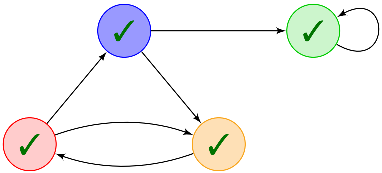

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[EH86] Emerson, Halpern. "Sometimes" and "Not Never" Revisited: On Branching versus Linear Time Temporal Logic. J.ACM, 1986.
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Reasoning about open systems

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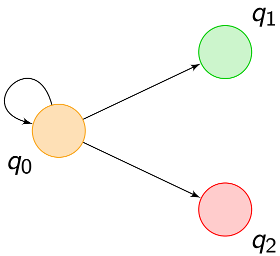
Concurrent games

A concurrent game is made of

- a transition system;



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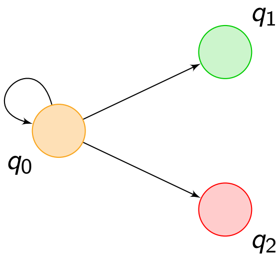

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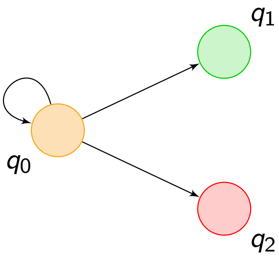

		player 1		
				
player 2		q_0	q_2	q_1
		q_1	q_0	q_2
		q_2	q_1	q_0

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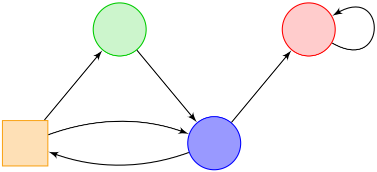
Turn-based games

A turn-based game is a game where only one agent plays at a time.



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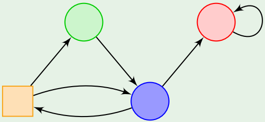
Strategy for player :
alternately go to  and .



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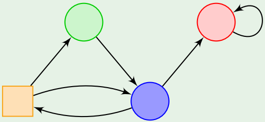

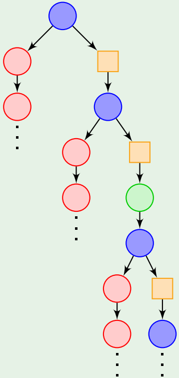

Temporal logics for games: ATL

ATL extends CTL with strategy quantifiers

$\langle\!\langle A \rangle\!\rangle\, \varphi$ expresses that A has a strategy to enforce φ.

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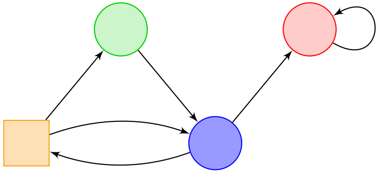

- $\langle\langle \bigcirc \rangle\rangle\ \mathbf{F}\ \bigcirc$

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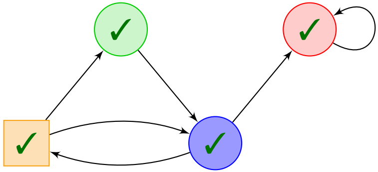

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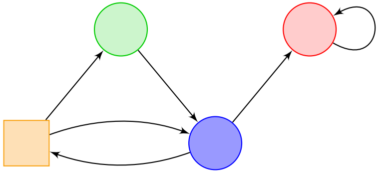

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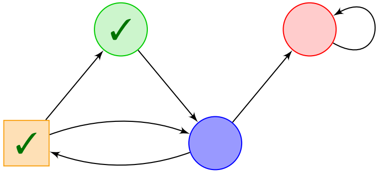

- $\langle\!\langle \bigcirc \rangle\!\rangle$ **F** ○
- $\langle\!\langle \Box \rangle\!\rangle$ **F** ○

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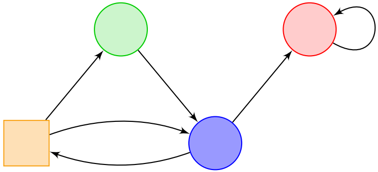

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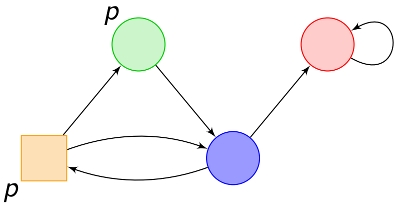

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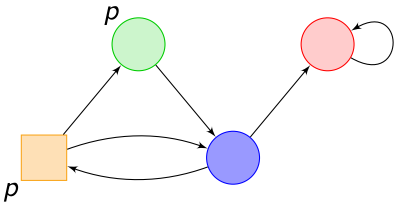

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Theorem ([AHK02])

Model checking ATL is PTIME*-complete.*
Model checking ATL* is 2-EXPTIME*-complete.*

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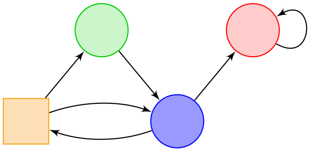
Outline of the presentation

ATL with strategy contexts [BDLM09]



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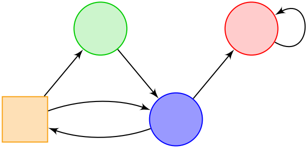

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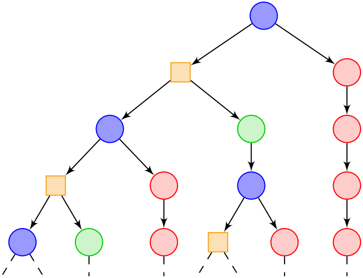

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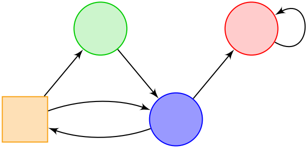

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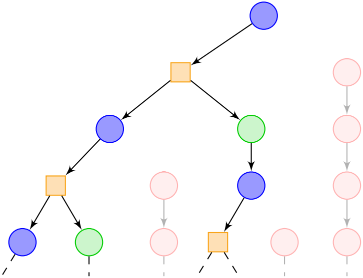

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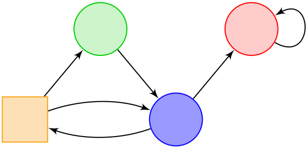

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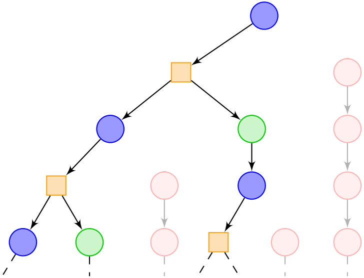

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- in the remaining tree, Player □ can always enforce a visit to ○.

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ATL with strategy contexts

Definition

ATL_{sc} has two new strategy quantifiers: $\langle\cdot A\cdot\rangle\,\varphi$ and $\langle\text{-}A\text{-}\rangle\,\varphi$.

- $\langle\cdot A\cdot\rangle$ is similar to $\langle\!\langle A\rangle\!\rangle$ but assigns the corresponding strategy to A for evaluating φ;

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- $\langle\text{-}A\text{-}\rangle$ drops the assigned strategies for A.
- $[\cdot A\cdot]$ is dual to $\langle\cdot A\cdot\rangle$:

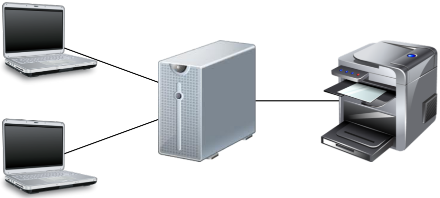
$$[\cdot A\cdot]\,\varphi \equiv \neg\,\langle\cdot A\cdot\rangle\,\neg\varphi$$

  $[\cdot A\cdot]\,\varphi$ which states that any strategy for A has an outcome along which φ holds.

What ATL_{sc} can express

- Client-server interactions for accessing a shared resource:

$$\langle\cdot \mathrm{Server}\cdot\rangle\ \mathbf{G} \left[\begin{array}{l} \displaystyle\bigwedge_{c\in \mathrm{Clients}} \langle\cdot c\cdot\rangle\ \mathbf{F}\, \mathrm{access}_c \\ \wedge \\ \displaystyle\neg \bigwedge_{c\neq c'} \mathrm{access}_c \wedge \mathrm{access}_{c'} \end{array} \right]$$



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- Existence of Nash equilibria:

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$$\langle\cdot A \cdot\rangle\ [\cdot B \cdot] \left(\neg \varphi \Rightarrow [\cdot A \cdot]\, \neg \varphi \right)$$

More expressiveness results

Theorem

- *ATL_{sc} is strictly more expressive than ATL,*
- *The operator $\langle\!\cdot A \cdot\!\rangle$ does not add expressive power,*
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$$\langle\!\langle A \rangle\!\rangle\, \varphi \equiv \langle\!\text{-}\mathrm{Agt}\text{-}\!\rangle\ \langle\cdot A \cdot\rangle\ \hat{\varphi}$$

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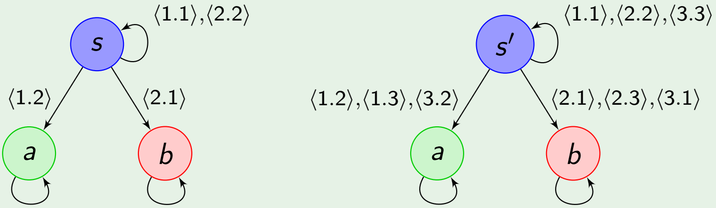
Proof

$\langle\cdot 1\cdot\rangle\,(\,\langle\cdot 2\cdot\rangle\ \mathbf{X}\ a \wedge\ \langle\cdot 2\cdot\rangle\ \mathbf{X}\ b)$ is only true in the second game.
But ATL cannot distinguish between these two games.



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Replace implicit quantification with explicit one:

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$\leadsto$ we can always assume that the context is full.

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$\rightsquigarrow$ we can always assume that the context is full.

- $\langle\!\cdot A \cdot\!\rangle\, \varphi$ is then equivalent to $[\cdot A \cdot]\, \langle\!\cdot \emptyset \cdot\!\rangle\, \varphi$;
- $\langle\!\cdot \emptyset \cdot\!\rangle$ can be inserted between two temporal modalities.

Outline of the presentation

1. Introduction
2. Basics of CTL and ATL
   - expressing properties of reactive systems
   - efficient verification algorithms
3. Temporal logics for multi-agent systems
   - specifying properties of complex interacting systems
   - expressive power of ATL_{sc}
   - translation into Quantified CTL (QCTL)
   - algorithms for ATL_{sc}
4. Conclusions and future works

Quantified CTL [ES84,Kup95,Fre01]

QCTL extends CTL with propositional quantifiers

$\exists p.\ \varphi$ means that there exists a labelling of the model with p under which φ holds.

[ES84] Emerson and Sistla. Deciding Full Branching Time Logic. Information & Control, 1984.
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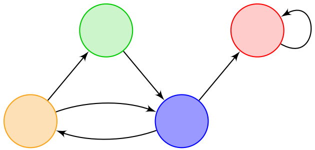

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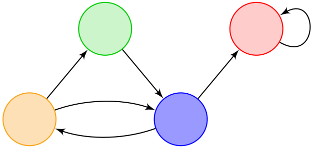

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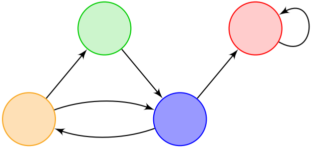

$\rightsquigarrow$ true if we label the Kripke structure;

$\rightsquigarrow$ false if we label the computation tree;

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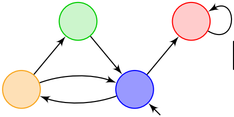
Semantics of QCTL

- structure semantics:



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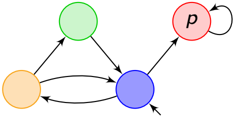

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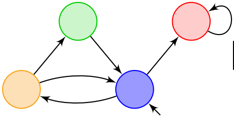

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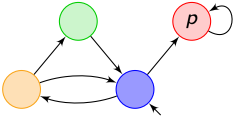

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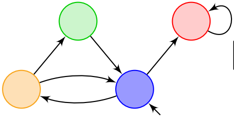

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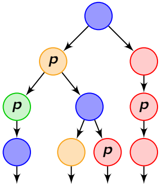

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Expressiveness of QCTL

- QCTL can "count":

$$\mathbf{E}\,\mathbf{X}_1\,\varphi \equiv \mathbf{E}\,\mathbf{X}\,\varphi \wedge \forall p.\ [\mathbf{E}\,\mathbf{X}(p \wedge \varphi) \Rightarrow \mathbf{A}\,\mathbf{X}(\varphi \Rightarrow p)]$$
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- QCTL can express (least or greatest) fixpoints:

$$\mu T.\varphi(T) \equiv \exists t.\ [\mathbf{A}\,\mathbf{G}(t \iff \varphi(t)) \wedge (\forall t.'(\mathbf{A}\,\mathbf{G}(t' \iff \varphi(t')) \Rightarrow \mathbf{A}\,\mathbf{G}(t \Rightarrow t')))]$$

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Theorem

QCTL, QCTL* and MSO are equally expressive (under both semantics).

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

QCTL with structure semantics

Theorem

Model checking QCTL for the structure semantics is PSPACE*-complete.*

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Proof

Membership:

Iteratively
- (nondeterministically) pick a labelling,
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Hardness:

QBF is a special case (without even using temporal modalities).

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QCTL satisfiability for the structure semantics is undecidable.

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- *Model checking QCTL with k quantifiers in the tree semantics is* k-EXPTIME*-complete.*
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[LM13a] Laroussinie, M. Quantified CTL: expressiveness and complexity. Submitted, 2013.

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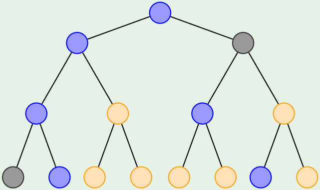

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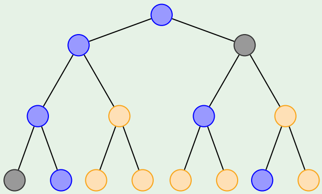

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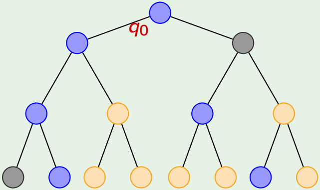

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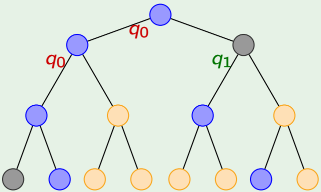

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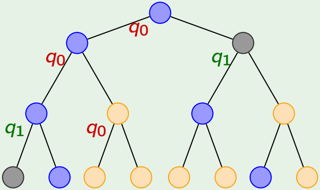

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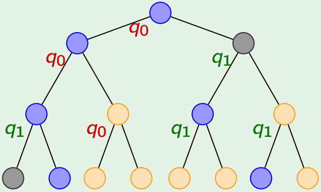

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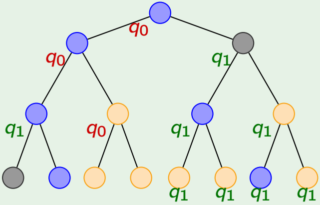

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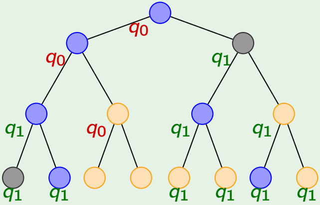

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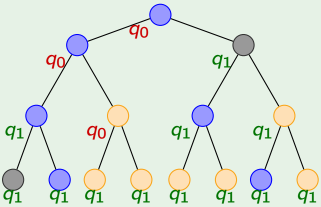

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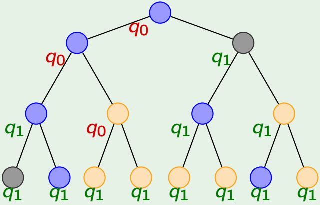

This automaton corresponds to **E**$\textcolor{blue}{\bullet}$ **U** $\textcolor{orange}{\bullet}$

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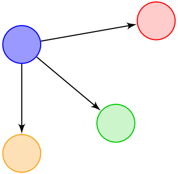
Proof

- polynomial-size automata for CTL;
- quantification is handled by projection, which first requires removing alternation (exponential blowup);
- an automaton equivalent to a QCTL formula can be built inductively;
- emptiness of an alternating parity tree automaton can be decided in exponential time.

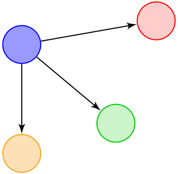
Translating ATL$_{sc}$ into QCTL



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(, A, m_i^A) of states that can be reached from  when player A plays m_i^A.

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

Translating ATL$_{sc}$ into QCTL



- player A has moves m_1^A, ..., m_n^A;
- from the transition table, we can compute the set Next(, A, m_i^A) of states that can be reached from when player A plays m_i^A.

$\langle\cdot A\cdot\rangle\,\varphi$ can be encoded as follows:

$\exists m_1^A.\ \exists m_2^A \ldots \exists m_n^A.$

- this corresponds to a strategy: $\quad \mathbf{A}\,\mathbf{G}(m_i^A \Leftrightarrow \bigwedge \neg\, m_j^A)$;
- the outcomes all satisfy φ:

$$\mathbf{A}\big[\mathbf{G}(q \wedge m_i^A \Rightarrow \mathbf{X}\,\mathrm{Next}(q, A, m_i^A)) \Rightarrow \varphi\big].$$

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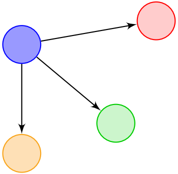

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- from the transition table, we can compute the set Next(, A, m_i^A) of states that can be reached from when player A plays m_i^A.

Corollary

*ATL$_{sc}$ model checking is decidable, with non-elementary complexity (*TOWER*-complete).*

Corollary

ATL$_{sc}^0$ (quantification restricted to memoryless strategies) model checking is PSPACE*-complete.*

[DLM12] Da Costa, Laroussinie, M. Quantified CTL: expressiveness and model checking. CONCUR, 2012.

What about satisfiability?

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QCTL satisfiability is decidable (for the tree semantics).

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ATL_{sc} satisfiability is undecidable.

Why?

The translation from ATL_{sc} to QCTL assumes that the game structure is given!

[TW12] Troquard, Walther. On Satisfiability in ATL with Strategy Contexts. JELIA, 2012.

Satisfiability for turn-based games

Theorem (LM13b)

When restricted to turn-based games, ATL_{sc} satisfiability is decidable.

[LM13b] Laroussinie, M. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

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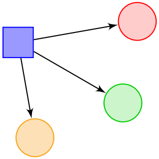

- player $\square$ has moves green, orange and red.
- a strategy can be encoded by marking some of the nodes of the tree with proposition mov_A.

$\langle\cdot A\cdot\rangle\ \varphi$ can be encoded as follows:

$\exists \mathsf{mov}_A.$

- it corresponds to a strategy: $\mathbf{A}\,\mathbf{G}(\mathsf{turn}_A \Rightarrow \mathbf{E}\,\mathbf{X}_1\, \mathsf{mov}_A)$;
- the outcomes all satisfy φ: $\mathbf{A}\big[\mathbf{G}(\mathsf{turn}_A \wedge \mathbf{X}\,\mathsf{mov}_A) \Rightarrow \varphi\big]$.

[LM13b] Laroussinie, M. Satisfiability of ATL with strategy contexts. Gandalf, 2013.

What about Strategy Logic? [CHP07,MMV10]

Strategy logic

Explicit quantification over strategies + strategy assignement

Example

$$\langle\cdot A\cdot\rangle\,\varphi \equiv \exists\sigma_1.\mathrm{assign}(\sigma_1, A).\varphi$$

Strategy logic can also be translated into QCTL.

Theorem

- *Strategy-logic model-checking is decidable.*
- *Strategy-logic satisfiability is decidable when restricted to turn-based games.*

[CHP07] Chatterjee, Henzinger, Piterman. Strategy Logic. CONCUR, 2007.
[MMV10] Mogavero, Murano, Vardi. Reasoning about strategies. FSTTCS, 2010.

Conclusions and future works

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- QCTL is a powerful extension of CTL;
- it is equivalent to MSO over finite graphs and regular trees;
- it is a nice tool to understand temporal logics for games (ATL with strategy contexts, Strategy Logic, ...);

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Future directions

- Defining interesting (expressive yet tractable) fragments of those logics;
- Obtaining practicable algorithms.
- Considering randomised strategies.